Published by IOP Publishing for SISSA
Received: *September 5, 2008*
Accepted: *September 22, 2008*
Published: *October 20, 2008*

U(1) mediation of flux supersymmetry breaking

Thomas W. Grimm and Albrecht Klemm

Physikalisches Institut der Universität Bonn,
Nussallee 12, 53115 Bonn, Germany
E-mail: grimm@th.physik.uni-bonn.de, aklemm@th.physik.uni-bonn.de

Abstract: We study the mediation of supersymmetry breaking triggered by background fluxes in Type II string compactifications with $\mathcal{N} = 1$ supersymmetry. The mediation arises due to an U(1) vector multiplet coupling to both a hidden supersymmetry breaking flux sector and a visible D-brane sector. The required internal manifolds can be constructed by non-Kähler resolutions of singular Calabi-Yau manifolds. The effective action encoding the U(1) coupling is then determined in terms of the global topological properties of the internal space. We investigate suitable local geometries for the hidden and visible sector in detail. This includes a systematic study of orientifold symmetries of del Pezzo surfaces realized in compact geometries after geometric transition. We construct compact examples admitting the key properties to realize flux supersymmetry breaking and U(1) mediation. Their toric realization allows us to analyze the geometry of curve classes and confirm the topological connection between the hidden and visible sector.

Keywords: Flux compactifications, Supersymmetry Breaking, Superstring Vacua, Supersymmetric Effective Theories.

JHEP10(2008)077

Contents

1. Introduction

The embedding of the standard model and its supersymmetric extensions into string theory is of crucial importance in the study of string phenomenology. One promising arena for model building are Type II string compactifications with gauge theories realized on intersecting D-branes extending along the four non-compact dimensions [1]. Particularly appealing are scenarios for which the visible gauge theory and matter interactions can be localized within the internal space. Some crucial parts of the four-dimensional physics, such as the gauge group and matter content, are then determined by the local geometry near the intersecting standard model branes. This opens the door for concrete model building within a well-defined local framework [1−4].

In addition to the strings confined to the standard model branes, there will be string states which propagate through the bulk and interact with hidden sectors which are separated from the visible branes within the internal space. In particular, supersymmetry breaking occurring in a hidden sector can be mediated to the visible branes via such messengers. There are essentially two categories for the mediation of supersymmetry breaking. Firstly, it can be mediated via gauge theory degrees of freedom which arise from open strings in Type II compactifications [5, 6]. Secondly, the mediation can occur due to closed string modes with Planck mass suppressed couplings [6]. The latter category includes the anomaly mediation scenarios arising from the gravitational sector [7]. In contrast to the universal anomaly mediation, the contributions of gauge and Planck suppressed mediation typically depend on the geometry and distances probed by the messengers between the visible and hidden sector. Different geometric set-ups can lead to a domination of one or the other mediation mechanism.

In this paper we investigate a string theory embedding of a mediation mechanism involving a U(1) vector multiplet. If this vector multiplet is coupling to both the visible as well as some hidden supersymmetry breaking sector, it can serve as a messenger of the breaking. The phenomenological aspects of such scenarios depend on the role of the U(1) in the visible sector. Concrete proposals use the possible existence of additional U(1) vectors, known as Z' gauge bosons, which couple to the Standard Model [8]. Mediation of supersymmetry breaking involving such extra U(1) multiplets has been studied in various contexts in refs. [9, 10]. Another recent proposal is to identify the mediating U(1) with the hypercharge of the MSSM [11]. Viable soft supersymmetry breaking terms are generated if the bino gains a mass from supersymmetry breaking in the hidden sector in addition to the anomaly mediation contribution to all soft terms. Set-ups with soft terms induces by U(1) mediation or anomaly mediation exhibits a number of very attractive phenomenological features such as natural suppression of CP- and flavor violation and a solution of the μ problem.

JHEP10(2008)077

In this work we specify a concrete Type IIB string compactification with the appropriate U(1) couplings and mechanism to break supersymmetry. More precisely, we argue that supersymmetry breaking due to non-vanishing R-R and NS-NS background fluxes can be naturally mediated by U(1) vector multiplets. The messengers arise as linear combinations of hidden and visible sector $\mathrm{U(1)_H}$ and $\mathrm{U(1)_V}$ vector multiplets. The coupling of the $\mathrm{U(1)_H}$ and $\mathrm{U(1)_V}$ is obtained from a gauging of a R-R scalar, similar to mechanism recently studied in ref. [12] applying the idea of [13]. However, in our set-ups the hidden $\mathrm{U(1)_H}$ gauge fields arise from the R-R four-form and pair in the underlying $\mathcal{N} = 2$ theory with the complex structure deformations of the internal manifold into supermultiplets. Exactly these scalars obtain an F-term in a non-supersymmetric flux background [14], and therefore render the U(1) gauginos massive. We show that simple topological relations connecting the visible and hidden sector within the internal space will ensure the generation of soft supersymmetry breaking terms for the standard model fields. In particular, we discuss how candidate internal geometries are constructed as non-Kähler resolutions of a singular Calabi-Yau manifolds.

Supersymmetry breaking by background fluxes has been investigated since the advent of flux compactification [15]. In particular, soft supersymmetry breaking terms induced by Planck suppressed moduli mediation have been first computed in refs. [16]. It thus has to be argued that the U(1) mediation will lead to a visible effect on the low energy masses. In general, direct couplings can be suppressed if the hidden supersymmetry breaking flux sector is separated, or rather 'sequestered' [7], from the standard model branes. This will be the case for set-ups where supersymmetry is broken by fluxes near a deformed singularity hidden in a warped throat away from the visible sector [18]. This does however still permit that the soft terms are corrected due to anomaly mediation, which yields to a mixture of two contributions as in the scenarios of refs. [17, 11]. Simple supersymmetry breaking flux backgrounds on a deformed Calabi-Yau singularity have been constructed in refs. [19], and argued to be large-N dual to meta-stable systems of $D5$ and anti-$D5$ branes. The orientifold versions of these models can serve as a hidden sector in our compactifications. Even though our analysis is more general and explicit, specific flux vacua will realize the large N dual of the U(1) mediation scenario of ref. [12].

In the construction of compact examples we need to ensure that both a visible brane sector as well as a hidden supersymmetry breaking sector can be realized. Geometrically this is achieved by picking Calabi-Yau manifolds with the appropriate singularities. The singularities are then resolved or deformed to yield a smooth compact internal manifold permitting U(1) mediation. In particular, this will lead us to the study of del Pezzo singularities and their resolutions. A del Pezzo surface is of real dimension 4 and has a sufficiently substructure to support intersecting branes inducing a spectrum and gauge group of a MSSM like model [3]. We will focus on the del Pezzo surfaces which are obtained by blowing up $\mathbb{P}^2$ at $5, 6, 7$ or 8 points. The blow up process and the specification of appropriate orientifold projections will be described in detail.

In order to find an explicit realization of the U(1) scenario, we have to be able to check the topological connection between the hidden and visible sector. We will argue that this is possible by analyzing the global embedding of the del Pezzo surface into the compact

JHEP10(2008)077

space. The computation of the BPS invariants for concrete torically realized examples reveals a decisive criterion to analyze which of the two-cycles in the del Pezzo surface are non-trivial in the compact space and connected to the hidden sector. Since the geometry of del Pezzo surfaces is captured by Lie algebras, this criterion can be reformulated in terms of the decomposition of Lie algebra representations.

This paper is organized as follows. In section 2 the general mechanism of U(1) mediated supersymmetry breaking and our sting theory embedding is summarized. This includes in section 2.1 a brief overview of the phenomenological properties of the scenarios suggested in refs. [10, 11]. In section 3 we discuss the basics on constructing the necessary internal geometries as well as the corresponding $\mathcal{N} = 1$ four-dimensional effective theories. The relevant non-Kähler resolutions are introduced in section 3.1. The $\mathcal{N} = 1$ orientifold projection as well as details on the effective action are presented in section 3.2. The candidate visible sectors arise from del Pezzo surfaces as discussed in section 3.3, while the hidden sector flux geometry inducing the supersymmetry breaking are studied in section 3.4.

In the second part of this work we turn to explicit geometrical constructions of the outlined set-up. The hidden geometry are orientifolds of A_n singularities introduced in section 4.2. More effort is devoted to the study of del Pezzo transitions in compact Calabi-Yau spaces. We analyze a large class of candidate internal manifolds in section 4.3 and 4.4, and discuss their orientifold symmetries. A compact manifolds with associated orientifold projection admitting most of the desired properties to permit U(1) mediation is constructed in section 5. The toric construction of the compact geometries as well as further explicit examples are provided in appendices A and B.

2. U(1) mediation of supersymmetry breaking

In this section we first describe the general mechanism how a U(1) vector multiplet coupling to both a hidden as well as a visible sector can mediate supersymmetry breaking. We briefly discuss in section 2.1 the phenomenological implications in case this U(1) coupling is one of the dominant mediation mechanisms. The general idea how to realize such a scenario within a flux compactification of type IIB string theory will be presented in section 2.2.

2.1 The mediation mechanism and its phenomenology

Let us consider a four-dimensional $\mathcal{N} = 1$ supersymmetric theory consisting of a visible MSSM-like sector with Lagrangian $\mathcal{L}_{\text{visible}}$ and a hidden sector $\mathcal{L}_{\text{hidden}}$. We denote the bosonic components of the supermultiplets in the visible sector collectively by Q, while they are denoted by Φ for the hidden sector. Supersymmetry breaking is assumed to take place in the hidden sector. The U(1) mediation of this breaking is possible if the field dependence of the effective low energy Lagrangian is of the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{visible}}(Q, A) + \mathcal{L}_{\text{hidden}}(\Phi, A) \,, \tag{2.1}$$

where A is a U(1) gauge boson in a vector multiplet coupling to both the visible and hidden sector. Generically both the hidden and visible sector will contain fields charged under A.

The holomorphic gauge kinetic coupling function of A will be denoted by $f(\phi)$ and will depend on the hidden sector chiral multiplets collectively denoted by ϕ.

Consider now supersymmetry breaking by a non-vanishing F-term in the hidden sector. The coupling of A to the hidden sector can yield a significant contribution $\tilde{M}$ to the mass of the fermion λ in the vector multiplet (A, λ). Recall that in an $\mathcal{N} = 1$ supersymmetric theory, the gauge kinetic term for the superfield $\mathcal{W}$ containing (A, λ) is given by

$$\mathcal{L}_{\rm kin}(W) = \int d\theta^2 \frac{1}{4} f(\phi)\, \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c. \ . \tag{2.2}$$

The F-terms in the hidden sector are denoted by F_I, F^I and take the form

$$F_I = D_{\phi^I} W \,, \qquad\qquad F^I = e^{K/2} K^{I\bar{J}} \bar{F}_{\bar{J}} \ . \tag{2.3}$$

Here ϕ^I are complex scalars in the chiral multiplets, W is the holomorphic superpotential, and $K, K^{I\bar{J}}$ are the Kähler potential and inverse Kähler metric. It follows from (2.2) that the fermion λ in the multiplet of the vector A acquires a mass

$$\tilde{M} = F^I \partial_{\phi^I} \log(\mathrm{Re} f) \ . \tag{2.4}$$

The presence of the massive fermion λ coupling to the visible sector can have a profound impact on the observed supersymmetry breaking phenomenology.

In a generic string compactification with non-vanishing F-terms also other mediation mechanism can contribute to the soft parameters in the visible sector. Clearly, it needs to be ensured that additional gauge interactions between the two sectors are subdominant. More severely, gravity mediation can contribute to the soft parameters of order F_I/M_P. In the string compactifications we will consider later, these contributions can be suppressed due to sequestering [18]. The dominant supergravity contribution then arises from anomaly mediation. In section 2.1.1 a mixing of U(1) mediation with anomaly mediation is illustrated for the specific model proposed in ref. [11].

To illustrate the phenomenological implications of U(1) mediation we will briefly review two recently proposed scenarios. In the first scenario the vector A is the hypercharge $\mathrm{U}(1)_\mathrm{Y}$ of the MSSM [11], while in the second scenario it corresponds to an additional $\mathrm{U}(1)'$ under which all MSSM particles are charged [8 − 10]. This overview is also meant to highlight the generic features which are eventually demanded from a string realization.

2.1.1 Hypercharged anomaly mediation

In reference [11] it was proposed to identify A with the hypercharge $\mathrm{U}(1)_\mathrm{Y}$ of the MSSM. It was assumed that the only source of supersymmetry breaking is a non-vanishing hidden sector F-term in (2.4) and the contributions due to anomaly mediation [7]. The mass $\tilde{M}$ in (2.4) will contribute to the bare bino mass. This is the decisive change of boundary conditions fed into the bino renormalization group equation running from the compactification scale to low energies. The breaking scale of anomaly mediation is set by the gravitino mass $m_{3/2} = e^{K/2}|W|$. The gaugino masses M_1, M_2, M_3, the soft masses m_i as well as the

Yukawa couplings A_{ijk} in the visible sector are then given by

$$\begin{aligned} M_1 &= \tilde{M} + \frac{b_1 g_1^2}{8\pi^2} m_{3/2}\,, & M_a &= \frac{b_a g_a^2}{8\pi^2} m_{3/2}, \quad a = 2,3\,, \\ m_i^2 &= -\frac{1}{32\pi^2}\frac{d\gamma_i}{d\log\mu} m_{3/2}\,, & A_{ijk} &= -\frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2} m_{3/2}\,, \end{aligned} \tag{2.5}$$

where b_a are the β-function coefficients, and γ_i are the anomalous dimensions of the matter fields. The g_i are the three gauge couplings of the MSSM.

The difference of the soft terms (2.5) to the ones arising in general gauge mediation scenarios [5] is that the supersymmetry breaking in the hidden sector only contributes at leading order to the bino mass M_1. The other soft masses are identical to the ones obtained for the anomaly mediation scenario. It was shown in ref. [11], that only for certain values of $\tilde{M}$ the renormalization group equation flow yields an acceptable low energy spectrum,

$$\tilde{M} \equiv \alpha\, m_{3/2}\,, \qquad 0.05 \lesssim |\alpha| \lesssim 0.25\,, \tag{2.6}$$

for $m_{3/2} \gtrsim 35\,\text{TeV}$. In an explicit string realization of this scenario the supersymmetry breaking mechanism in the hidden sector has to induce F-terms generating an $\tilde{M}$ satisfying the bound (2.6).

2.1.2 Mediation by an additional U(1)$'$

In this section we briefly review a scenario where U(1) mediation of supersymmetry breaking arises due to an additional U(1)$'$ factor extending the MSSM gauge group [8 − 10]. In references [10] it was proposed to extend the MSSM by an additional U(1)$'$ gauge symmetry under which all MSSM fields, as well as a new Standard Model singlet S are charged. Here S replaces the μ parameter of the MSSM. This extended MSSM needs to include a number of exotics with Yukawa couplings to S in order to cancel anomalies [10]. From the point of view of string theory, the presence of additional U(1) symmetries and exotics is rather generic and models such as the one presented in [10] might therefore admit a natural embedding into a string compactification.

In the scenario of [10] it was demanded that the U(1)$'$ gauge symmetry is not broken in the hidden sector, but rather in the visible sector through a vev of the additional field S. In contrast, supersymmetry breaking is assumed to take place in the hidden sector. The non-vanishing F-terms generate a mass $\tilde{M}$ for the fermion in the U(1)$'$ vector multiplet directly as in (2.4) or through loop corrections involving the supersymmetry breaking scalars. For a Lagrangian of the form (2.1) there are no direct couplings to the hidden sector and supersymmetry breaking is mediated by the U(1)$'$. Since all chiral multiplets are charged under the U(1)$'$ the fermion soft masses m_i are generated already by a one loop correction. The gauginos do not directly couple to the U(1)$'$ and are thus only induced at the two loop level. Explicitly, the gaugino and scalar soft masses take the form

$$M_a \sim \frac{\tilde{g}^2 g_a^2}{(16\pi^2)^2}\tilde{M}\ \log\left(\frac{\Lambda_S}{\tilde{M}}\right), \qquad m_i^2 \sim \frac{\tilde{g}^2 Q_i^2}{16\pi^2}\tilde{M}^2\ \log\left(\frac{\Lambda_S}{\tilde{M}}\right), \tag{2.7}$$

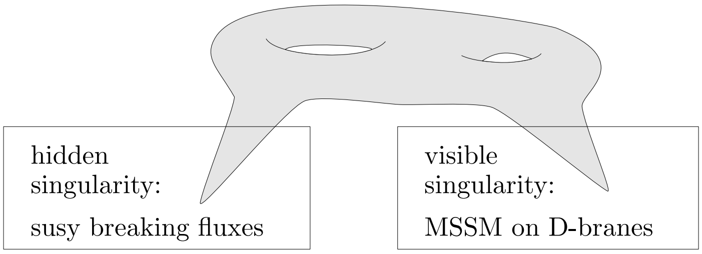

Figure 1: Compact manifold with hidden and visible sector singularity.

where $\tilde{g}$ is the $\mathrm{U}(1)'$ gauge coupling, Q_i are the $\mathrm{U}(1)'$ charges of the matter multiplets, and Λ_S is the scale of supersymmetry breaking. We refer the reader to ref. [10] for a detailed analysis of the phenomenology of this model.

In this work we will study the necessary steps for embedding U(1) models such as the ones of section 2.1.1 and 2.1.2 in a Type IIB flux compactification. This requires a precise specification of the supersymmetry breaking hidden sector and its $\mathrm{U}(1)'$ coupling to the visible sector.

2.2 An embedding into string theory

In the following we will describe a string theory scenario which admits a four-dimensional low energy effective Lagrangian of the form (2.1). We will consider a type IIB compactification on a manifold admitting two, possibly warped, singularities as schematically depicted in figure 1. The visible singularity is resolved by two- or four-dimensional cycles while the hidden singularity is deformed by three-cycles. If the internal manifold obeys certain topological conditions, we argue that such a set-up allows a U(1) mediation of supersymmetry breaking triggered by background R-R and NS-NS three-form fluxes to a visible sector.

2.2.1 Mixing hidden and visible U(1) vector multiplets

The string compactifications of interest admit two U(1) gauge fields A_{H} and A_{V} coupling to the hidden and the visible sector respectively. Their kinetic terms in an $\mathcal{N}=1$ effective theory are of the form

$$\mathcal{L}_{\mathrm{kin}} = \sum_{i=\mathrm{V,H}} \left(\frac{1}{2}\mathrm{Re} f_i\, F_i \wedge *F_i + \frac{1}{2}\mathrm{Im} f_i\, F_i \wedge F_i \right), \tag{2.8}$$

where $F_{\mathrm{V}} = dA_{\mathrm{V}}$, $F_{\mathrm{H}} = dA_{\mathrm{H}}$, and f_{V}, f_{H} are the holomorphic gauge-coupling functions. In the following we will review the mechanism suggested in ref. [13, 12] to obtain an effective Lagrangian (2.1) for a linear combination A of A_{H} and A_{V}. Namely, as we will check for our scenario later on, the dimensional reduction to four space-time dimensions can induce a coupling term of the form

$$\mathcal{L}(\mathcal{C}) = \mathcal{C} \wedge d(q\, A_{\mathrm{V}} + e\, A_{\mathrm{H}}) + \frac{1}{2\mu^2}|d\mathcal{C}|^2\,, \tag{2.9}$$

where $\mathcal{C}$ is a massless two-form field and e, q are U(1) charges. In four space-time dimensions the massless two-form $\mathcal{C}$ is dual to a scalar ρ. The effective Lagrangian obtained by dualizing (2.9) is given by

$$\mathcal{L}(\rho) = \frac{1}{2}\mu^2 |d\rho + q\, A_{\rm V} + e\, A_{\rm H}|^2 \ . \tag{2.10}$$

As a consequence of the standard Higgs mechanism for ρ there is a heavy and a massless mass eigenstate

$$A^{\rm h} = q\, A_{\rm V} + e\, A_{\rm H}\,, \qquad\qquad q\, A = q\, A_{\rm V} - e\, A_{\rm H}\ . \tag{2.11}$$

Here we normalized the light U(1) such that the visible sector fields have the same charge under $2A$ and $A_{\rm V}$. To make the Higgsing more explicit, we note that in an $\mathcal{N} = 1$ supersymmetric theory the scalar ρ in (2.10) will combine with a second real scalar v to form the bosonic content of a chiral multiplet. This chiral multiplet is then absorbed by the U(1) vector multiplet $A^{\rm h}$ to form a massive vector multiplet $(A^{\rm h}, \xi(v))$, where $\xi(v)$ is the dynamical Fayet-Iliopoulos term. The scalar ρ has been absorbed by the gauge transformation $A^{\rm h} \to A^{\rm h} - d\rho$ in (2.10). In a string compactification the heavy mass state $A^{\rm h}$ has typically a mass of order string scale and can be integrated out. The resulting effective Lagrangian takes the desired form (2.1). Up to corrections suppressed by the mass of $A^{\rm h}$, the effective gauge-coupling of the massless U(1) is given by

$$4{\rm Re}f = {\rm Re}f_{\rm V} + (q/e)^2\ {\rm Re}f_{\rm H}\ . \tag{2.12}$$

The effective gaugino mass $\tilde{M}$ for the light U(1) is computed by inserting (2.12) into (2.4). Note that the above results are only true at leading order, and further corrections will arise due to integrating out the massive U(1) vector multiplet. The effects on the pattern of soft supersymmetry breaking terms can be evaluated rather general as shown in ref. [20], following earlier works [21].

2.2.2 Outline of the scenario

To be more explicit we now consider type IIB string theory compactified on a six-dimensional manifold $\mathcal{M}_6$, which we choose to be a non-Kähler deformation of a Calabi-Yau 3-fold, such that the four-dimensional effective theory is still an $\mathcal{N} = 2$ supergravity theory. The supersymmetry will be further reduced to $\mathcal{N} = 1$ by an orientifold projection and the inclusion of space-time filling D-branes [15]. For many orientifold projections the spectrum of this theory contains a number of U(1) vector multiplets with vectors arising from the R-R four-form C_4. For simplicity, let us concentrate on one such vector field $A_{\rm H}$ and its magnetic dual $\tilde{A}_{\rm H}$. Both arise as Kaluza-Klein modes of C_4 by integrating

$$A_{\rm H} = \int_{\mathcal{A}} C_4\,, \qquad\qquad \tilde{A}_{\rm H} = \int_{\mathcal{B}} C_4\,, \tag{2.13}$$

where $\mathcal{A}, \mathcal{B}$ are three-dimensional submanifolds in $\mathcal{M}_6$ with $\mathcal{A} \cap \mathcal{B} = 1$. That C_4 contains both $A_{\rm H}$ and $\tilde{A}_{\rm H}$ is due to the fact that its field strength $F_5 = d_{10} C_4$ needs to obey the ten-dimensional self-duality constraint $F_5 = *F_5$. For appropriate $\mathcal{A}, \mathcal{B}$ the self-duality of

F_5 implies the electro-magnetic duality between $A_{\rm H}$ and $\tilde{A}_{\rm H}$. As indicated by the notation, the vector $A_{\rm H}$ will correspond to the hidden sector $\mathrm{U}(1)_{\rm H}$ of section 2.1.

The visible MSSM-like sector is realized on stacks of space-time filling D3 or D7 branes [1−4]. Later on, we will mostly focus on intersecting branes on a del Pezzo four-cycle S. The resulting four-dimensional effective gauge theory generically contain a number of U(1) factors which arise, for example, from the splitting of the U(N) gauge groups on a stack of N branes into $\mathrm{U}(N) = \mathrm{SU}(N) \times \mathrm{U}(1)$. An appropriate combination of such U(1) factors will provide a vector field $A_{\rm V}$, the visible sector $\mathrm{U}(1)_{\rm V}$. In the MSSM $\mathrm{U}(1)_{\rm V}$ has to coincide with the non-anomalous hypercharge when modeling the scenario of section 2.1.1. In many intersecting brane models also additional U(1) symmetries are induced and can be identified with the $\mathrm{U}(1)'$ of the model in section 2.1.2.

So far we introduced two decoupled sectors containing $A_{\rm H}$ and $A_{\rm V}$ respectively. In order to obtain an effective action of the form (2.1), with a common light U(1) vector field $A = qA_{\rm V} - eA_{\rm H}$, our string compactification should admit the couplings in the Stückelberg Lagrangian (2.9) or equivalently (2.10) to a four-dimensional two-form $\mathcal{C}$ or its dual scalar ρ. In our orientifold compactification, both $\mathcal{C}$ and ρ are obtained as Kaluza-Klein modes of the R-R four-form C_4 as

$$\mathcal{C} = \int_\Sigma C_4 \; . \qquad\qquad \rho = \int_{\tilde{\Sigma}} C_4 \; . \tag{2.14}$$

Here Σ and $\tilde{\Sigma}$ are two- and four-dimensional submanifolds of $\mathcal{M}_6$ respectively, fulfilling $\Sigma \cap \tilde{\Sigma} = 1$. The self-duality of F_5 implies the four-dimensional duality of ρ and $\mathcal{C}$.

In the next step we have to specify the topology of $\mathcal{M}_6$ and the properties of $\mathcal{A}, \mathcal{B}$ in (2.13) as well as $\Sigma, \tilde{\Sigma}$ in (2.14) in order that $A_{\rm V} + A_{\rm H}$ is massive in our string compactification. Let us first discuss the gauge term of the form (2.10) induced for $A_{\rm H}$. A more detailed analysis of this gauging can be found in section 3. In Calabi-Yau reductions, where $\mathcal{A}, \mathcal{B}$ and $\Sigma, \tilde{\Sigma}$ are harmonic cycles, ρ and $\mathcal{C}$ do not couple to the vector field $A_{\rm H}$. However, we can couple ρ to $A_{\rm H}$ if we impose the topological conditions

$$\partial\tilde{\Sigma} = e\,\mathcal{A}\,, \qquad\qquad \partial\mathcal{B} = e\,\Sigma\,, \tag{2.15}$$

i.e. for some constant e the cycles $e\mathcal{A}$ and $e\Sigma$ should be boundaries of $\tilde{\Sigma}$ and $\mathcal{B}$ respectively. The kinetic terms for ρ arise from the ten-dimensional term $\int F_5 \wedge * F_5$. The first topological relation in (2.15) yields that

$$\int_{\tilde{\Sigma}} F_5 = d\rho + \int_{\partial\tilde{\Sigma}} C_4 = d\rho + e\,A_{\rm H} \; . \tag{2.16}$$

Here we split $d_{10} = d_4 + d_6$, applied Stokes Theorem in 6 dimensions and used (2.13) and (2.14). In other words, the condition (2.15) implies that the scalar ρ gets gauged by the $\mathrm{U}(1)_{\rm H}$ exactly as in (2.10). Note that (2.15) together with the fact that ρ sits in the same supermultiplet as a Kähler modulus implies that $\mathcal{M}_6$ cannot be a Kähler manifold as we show in section 3.

In a next step we also need to couple the vector field $A_{\rm V}$ to the R-R fields ρ or $\mathcal{C}$. Here it is natural to concentrate on the coupling to $\mathcal{C}$ via the Chern-Simons action of the

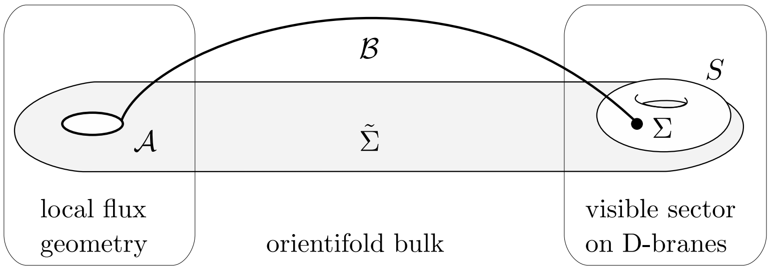

Figure 2: Non-Calabi-Yau space with two local geometries. The three-dimensional chain $\mathcal{B}$ reaching through the orientifold bulk has a two-dimensional boundary $\Sigma = \partial\mathcal{B}$ in the visible MSSM sector on a four-cycle S. The four-dimensional chain $\tilde{\Sigma}$ reaching through the bulk has a three-dimensional boundary $\mathcal{A} = \partial\tilde{\Sigma}$ in the hidden flux geometry.

space-time filling D7 branes. As we will discuss in section 3.3, this action contains a term of the form

$$\int_{\mathcal{W}_{7,1}} C_4 \wedge \mathcal{F} \wedge \mathcal{F} = q \int_{\mathbb{M}_{3,1}} \mathcal{C} \wedge dA_{\rm V} + \dots \,, \tag{2.17}$$

where $\mathcal{F}$ is the field strength on the D7 brane world-volume $\mathcal{W}_{7,1}$ and $\mathbb{M}_{3,1}$ is our space-time. Here q is an induced D5 charge arising from fluxes on the D7 brane. The coupling to $\mathcal{C}$ defined in (2.14) can be non-vanishing if Σ is in the world-volume of the D7 brane.[1] The coupling (2.17) is a Stückelberg mass term of the form (2.9). Since ρ and $\mathcal{C}$ are dual in four dimensions we can thus combine (2.16) and (2.17) showing that ρ is gauged as in (2.10). Precisely as in (2.11) this determines a massless and a heavy linear combination $A, A^{\rm h}$. A schematic overview of our set-up is presented in figure 2.

To complete our string set-up we now have to discuss how the fermion λ in the vector multiplet (A, λ) receives a bare mass from an hidden sector F-term as in (2.4). Here the key point is, that the gauge coupling function $f_{\rm H}$ of $A_{\rm H}$ is holomorphic in the complex structure deformations of $\mathcal{M}_6$ [28]. The complex structure deformations appear in the flux superpotential $W_{\rm flux}$ induced by R-R and NS-NS three-form fluxes F_3 and H_3. Denoting by τ the complex dilaton axion and by Ω the holomorphic three-form on $\mathcal{M}_6$ we have

$$W = \int_{\mathcal{M}_6} G_3 \wedge \Omega \,, \qquad G_3 = F_3 - \tau H_3 \,. \tag{2.18}$$

The superpotential depends holomorphically on the complex structure deformations through Ω. The corresponding scalar potential can admit minima with non-vanishing F-terms, which thus induce a bare mass for λ.

3. U(1) mediation in non-Kähler compactifications

In this section we study a detailed realization of U(1) mediated supersymmetry breaking in a Type IIB orientifold compactification. We discuss the construction of the internal non-Kähler geometries in section 3.1. The orientifold projection as well as the four-dimensional

[1] Strictly speaking, it will be sufficient if there exists a two-cycle in the world-volume of the D7 brane which can support D5 charge which induces the coupling (2.17). As for the compact orientifold example presented in section 5 this can be the case even though Σ itself is not a curve in the D7 world-volume.

$\mathcal{N}=1$ effective action and its characteristic functions are studied in section 3.2. The visible sector on a del Pezzo surface is introduced in section 3.3, while the hidden flux sector breaking supersymmetry is discussed in section 3.4. The main focus of this section is the study of the four-dimensional effective action in terms of the topological data and relations of the internal manifold. Most of the explicit geometric constructions are postponed to section 4.

3.1 Non-Kähler resolutions

Let us focus on the compactification of type IIB string theory on a six-dimensional manifold $\mathcal{M}_6$. In order to realize a U(1) coupling between the hidden flux and the visible brane sector as in section 2.2, $\mathcal{M}_6$ cannot be a Calabi-Yau manifold. However, in order that the four-dimensional effective theory still possesses some supersymmetry, $\mathcal{M}_6$ needs to admit a globally defined three-form Ω and a two-form J, which define an SU(3) structure on $\mathcal{M}_6$ such that

$$J \wedge \Omega = 0\,, \qquad J \wedge J \wedge J = c\,\Omega \wedge \bar{\Omega} \neq 0\,, \tag{3.1}$$

for some complex constant c on $\mathcal{M}_6$ [22 – 24]. Ω and J are the analogs of the holomorphic three-from and Kähler form on a Calabi-Yau manifold. However, on a general SU(3) structure manifold both dJ and $d\Omega$ can be non-vanishing.

In our set-up we wish to deviate as little as possible from the Calabi-Yau geometry in order to keep in good approximation the powerful calculational tools of the $\mathcal{N}=2$ special geometry. Therefore, we will restrict ourselves to manifolds which are still *complex*, but can be *non-Kähler*. In terms of J, Ω this implies [22]

$$d\Omega = 0\,, \qquad dJ = \mathcal{W}_3\,, \qquad \mathcal{W}_3 \wedge J = 0\,, \tag{3.2}$$

where $\mathcal{W}_3$ is the three-form parameterizing the obstruction of $\mathcal{M}_6$ being Kähler. The condition (3.2) will be realized by a non-Kähler resolution of a singular Calabi-Yau manifold [25 – 27].

Let us consider conifold transitions between a Calabi-Yau manifold Y to a manifold $\mathcal{M}_6$. In such a topological transition one or more cycles $\mathcal{A}_i$ of S^3 topology are shrunken to a node and resolved by exceptional two-cycles Σ_a with S^2 topology. There are global restrictions which need to be satisfied in order that $\mathcal{M}_6$ remains Kähler. A well-known example of Kähler transitions are transitions between Calabi-Yau manifolds $Y \rightarrow Y' = \mathcal{M}_6$. These occur if the shrinking three-cycles obey a number of relations in homology. For example, let us consider a transition in which k S^3's, denoted by $\mathcal{A}_i$, shrink to nodes which are subsequently blown up to k S^2's. Suppose δ is the number of homological relations

$$\sum_{i=1}^{k} c^i_j\ \mathcal{A}_i = \partial \tilde{\Sigma}_j\,, \qquad j = 1, \ldots, \delta\,, \tag{3.3}$$

with constant coefficients c^i_j and four-chains $\tilde{\Sigma}_j$. Since the independent A-cycles correspond locally one to one to variations of the complex structure, one has to fix $k-\delta$ complex structure moduli to create the k nodes in Y. This implies $h^{2,1}(Y) - h^{2,1}(Y') = k - \delta$.

JHEP10(2008)077

Figure 3: Divisor $\tilde{\Sigma}$ with three nodes p, p', q.

Further, in order for Y' to be Kähler with $dJ = 0$ there must be $k - \delta$ homology relations among the k exceptional S^2's. If this is the case a Calabi-Yau transition from Y to Y' exists and $h^{1,1}(Y') - h^{1,1}(Y) = \delta$. As we will argue next, one can also violate the Kähler condition in a simple and controlled way and thus construct non-Kähler manifolds.

Let us start with the simplest example of a transition to a non-Kähler manifold. From the above discussion, we infer that one can never shrink a single S^3 cycle $\mathcal{A}$, which is non-trivial in homology, i.e. $k = 1$, $\delta = 0$ in (3.3), and resolve the singular geometry by an S^2 cycle Σ such that the resulting geometry is Kähler. The reason is that the non-trivial three-cycle $\mathcal{B}$, which is symplectic dual to $\mathcal{A}$ with $\mathcal{A} \cap \mathcal{B} = 1$, develops a puncture at the nodal singularity as $\mathcal{A}$ shrinks to zero size. It is easily seen from the local geometry near the intersection of $\mathcal{A}, \mathcal{B}$ that the exceptional two sphere Σ becomes then a boundary of $\mathcal{B}$. As a consequence as soon as one resolves the node to Σ with finite size $v = vol(S^2)$ one gets [25, 26]

$$0 \neq v = \int_\Sigma J = \int_{\partial \mathcal{B}} J = \int_{\mathcal{B}} dJ \ . \tag{3.4}$$

The non-vanishing dJ implies that the manifold $\mathcal{M}_6$ cannot be Kähler.

These transitions to non-Kähler manifolds can be generalized to yield the set-ups suggested in section 2.2. A convenient way to achieve this is to both resolve and deform nodes located on a divisor $\tilde{\Sigma}$. Let us illustrate such a process on a simple example which can be generalized easily to more complicated situations. We assume that we have a singular Calabi-Yau manifold with three nodes p, p', q located on some divisor $\tilde{\Sigma}$ as depicted in figure 3. After deforming all singularities into three-spheres $\mathcal{A}, \mathcal{A}', \tilde{\mathcal{A}}$ they obey

$$\mathcal{A} + \mathcal{A}' + \tilde{\mathcal{A}} = \partial \tilde{\Sigma} \, , \tag{3.5}$$

which is (3.3) for $\delta = 1$ and $k = 3$. If (3.5) is the only condition in homology relating the $\mathcal{A}$'s there will exist two symplectic dual non-trivial three-cycles $\mathcal{B}, \mathcal{B}'$ with $\mathcal{A} \cap \mathcal{B} = \mathcal{A}' \cap \mathcal{B}' = 1$ and $\mathcal{A} \cap \mathcal{B}' = \mathcal{A}' \cap \mathcal{B} = 0$ such that

$$\mathcal{B} \cap (\mathcal{A} + \mathcal{A}' + \tilde{\mathcal{A}}) = \mathcal{B}' \cap (\mathcal{A} + \mathcal{A}' + \tilde{\mathcal{A}}) = 0 \ . \tag{3.6}$$

Clearly, we could also resolve any number of nodes into two-spheres. This corresponds to a geometric transitions of some or all of the three-cycles in (3.5). As discussed above, the blow-up process can preserve $dJ = 0$ if there are homological relations between the shrinking three-cycles, since the Kähler volumes of the resolving S^2's can cancel. However, we can also decide to only resolve the third node q into a two-sphere Σ, while deforming the nodes p, p' into three-spheres $\mathcal{A}, \mathcal{A}'$. This is shown in figure 4 and corresponds to a non-Kähler transition, with a homological relation that becomes very useful for our geometric engineering of supersymmetry breaking.

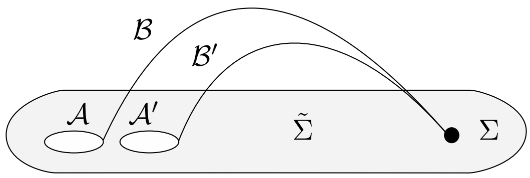

Figure 4: Divisor $\tilde{\Sigma}$ with boundary $\mathcal{A} + \mathcal{A}'$ and the three-chains $\mathcal{B}, \mathcal{B}'$ with boundary Σ.

The evaluation of dJ is now similar to (3.4). The condition (3.6) implies that the two $\mathcal{B}, \mathcal{B}'$ as well as $\tilde{\Sigma}$ have a boundary

$$\partial\mathcal{B} = \partial\mathcal{B}' = \Sigma\,, \qquad\qquad \partial\tilde{\Sigma} = \mathcal{A} + \mathcal{A}'\ . \tag{3.7}$$

Performing the integral of dJ over $\mathcal{B}, \mathcal{B}'$ we thus find

$$\frac{1}{2}\int_{\mathcal{B}+\mathcal{B}'} dJ = \int_{\Sigma} J = v\,, \qquad\qquad \int_{\mathcal{B}-\mathcal{B}'} dJ = 0\ . \tag{3.8}$$

This implies that dJ is non-vanishing and the resulting manifold $\mathcal{M}_6$ is non-Kähler. However, we can ensure that $d\Omega = 0$ on $\mathcal{M}_6$ by canceling the two holomorphic volumes of the deformed S^3's. More precisely, due to the second relation in (3.7) we need to obey the condition

$$0 = \int_{\tilde{\Sigma}} d\Omega = \int_{\mathcal{A}+\mathcal{A}'} \Omega = X^1 + X^2\,, \qquad\qquad X^1 = \int_{\mathcal{A}} \Omega\,, \quad X^2 = \int_{\mathcal{A}'} \Omega\,, \tag{3.9}$$

such that $X^1 = -X^2$. As we will see in section 3.2, this condition can be consistently imposed together with an orientifold involution of $\mathcal{M}_6$ which maps $\mathcal{A}$ to $\mathcal{A}'$.

Let us also comment on the construction of figure 4 from the point of view of the two local geometries. In order to do that we can imagine that we zoom into either of the two regions of figure 4. One region contains a patch around the small three-spheres $\mathcal{A}, \mathcal{A}'$, while the second region is the patch containing the two-sphere Σ. Effectively, in this process we obtain two non-compact geometries by scaling the connecting chains $\mathcal{B}, \mathcal{B}'$ and $\tilde{\Sigma}$ to be infinitely large. In the local geometries we are still able to identify the compact two- and three-cycles Σ and $\mathcal{A}, \mathcal{A}'$, while the information about the chains is lost in performing the local limit. In particular, this implies that the condition (3.8), (3.9) cannot be evaluated in the local geometry and require global information about $\mathcal{M}_6$. In non-compact geometries one has to *choose* the dual non-compact cycles $\tilde{\Sigma}, \mathcal{B}, \mathcal{B}'$ by fixing appropriate boundary conditions. When patched into a compact space one has to demand that they fulfill the conditions (3.7).

3.2 The effective action of non-Kähler orientifolds

Compactifying type IIB string theory on the non-Kähler manifold $\mathcal{M}_6$ leads to a four-dimensional effective $\mathcal{N} = 2$ supergravity theory [23]. An appropriate orientifold projection will reduce this further to $\mathcal{N}=1$ [29, 30]. For set-ups with O3/O7 planes the orientifold projection $\mathcal{O}=\Omega_p(-1)^{F_L}\sigma$ contains an involution σ obeying

$$\sigma^* J = J\,, \qquad\qquad \sigma^*\Omega = -\Omega\,, \tag{3.10}$$

where J and Ω are the globally defined two and three-form obeying (3.1) and (3.2). The orientifold symmetry σ also splits the cohomologies into positive and negative eigenspaces $H^n = H^n_+ \oplus H^n_-$ with dimensions $b^n_\pm = \dim H^n_\pm$. For simplicity, we will restrict to manifolds $\mathcal{M}_6$ with $b^2_- = 0$. To remain in the orientifold theory the ten-dimensional NS-NS B-field B_2 and the R-R forms C_0, C_2, C_4 have to transform under the involution σ as

$$\sigma^* B_2 = -B_2\,, \qquad \sigma^* C_p = (-1)^{p/2}\, C_p\ . \tag{3.11}$$

In performing the Kaluza-Klein reduction to four space-time dimensions, we have to expand all ten-dimensional fields in forms on $\mathcal{M}_6$ transforming with the appropriate sign under σ^*. Note that in a consistent compactification on a non-Kähler space this will also involve non-harmonic representatives [23, 29 − 31]. In the following we generalize the discussion of section 3.1 to the case of N non-Kähler resolutions. We denote the small resolving two-spheres by Σ_i, while the deforming pairs of three-spheres are denoted by $\mathcal{A}_i, \mathcal{A}'_i$, with $i=1,\ldots,N$. Later on, we will realize all Σ_i in a four-cycle $S\in H_4(\mathcal{M}_6)$ on which the visible gauge theory is modeled. In other words we will identify $\Sigma_i \in H_2(S)$, while there will be homological relations among the Σ^i within $\mathcal{M}_6$. The compact three-cycles $\mathcal{A}_i, \mathcal{A}'_i$ will support the hidden flux geometry. They are non-trivial in the local Calabi-Yau geometry around the hidden singularity. The chains connecting the two- and three-cycles in $\mathcal{M}_6$ are denoted by $\tilde\Sigma^i$ and $\mathcal{B}^i, \mathcal{B}'^i$. In order to connect the hidden and visible sector we demand that they obey

$$\partial(\mathcal{B}^i + \mathcal{B}'^i) = e^{ij}\Sigma_j\,, \qquad \partial\tilde\Sigma^i = e^{ij}(\mathcal{A}_j + \mathcal{A}'_j)\,, \tag{3.12}$$

for some constant matrix e^{ij} of rank N. The equation (3.12) is the generalization of (3.7). In summary, we associate to each non-Kähler resolution

$$(\Sigma_i, \tilde\Sigma^i)\,, \qquad (\mathcal{A}_i, \mathcal{A}'_i, \mathcal{B}^i, \mathcal{B}'^i)\,, \qquad i=1,\ldots,N\ . \tag{3.13}$$

The $\mathcal{N}=1$ orientifold involution σ^* introduced in (3.10) is chosen such that

$$\sigma^*\Sigma_i = \Sigma_i\,, \qquad \sigma^*\tilde\Sigma^i = \tilde\Sigma^i\,, \qquad \sigma^*\mathcal{A}_i = \mathcal{A}'_i\,, \qquad \sigma^*\mathcal{B}^i = \mathcal{B}'^i\,, \tag{3.14}$$

Note that the first two conditions are not necessarily true point-wise for all points on $\Sigma_i, \tilde\Sigma^i$, such that $\Sigma_i, \tilde\Sigma^i$ are not necessarily entirely inside an orientifold plane. Finally, there can be cycles $\Gamma_a \in H_2(S)$ in the visible sector region transforming as

$$\sigma^*\Gamma_a = -\Gamma_a\ . \tag{3.15}$$

Since we demand that $b^2_- = 0$ the cycles Γ_a will be trivial in $\mathcal{M}_6$. Note that a convenient way to invariantly characterize the introduced basis is provided by using relative homology.[2] For

[2] See e.g. [32] for an introduction to relative homology.

JHEP10(2008)077

chain	$\dim_{\mathbb{R}}$	relations	σ-parity
$(\Sigma_i, \mathcal{B}^i + \mathcal{B}'^i)$	$(2,3)$	$\partial(\mathcal{B}+\mathcal{B}') = e\Sigma$	$+$
$(\tilde{\Sigma}^i, \mathcal{A}_i + \mathcal{A}'_i)$	(4,3)	$\partial\tilde{\Sigma} = e(\mathcal{A}+\mathcal{A}')$	$+$
Γ_a	2	boundary	$-$

Table 1: Non-harmonic chains used in the Kaluza-Klein reduction.

example, the relative homology group $H_3(\mathcal{M}_6/S)$, will by definition contain the elements of $H_3(\mathcal{M}_6)$ as well as three-chains with boundaries on S. Using the chains introduced in (3.13) and (3.15) we will be able to determine the spectrum of the four-dimensional effective theory. It is then shown that the Kaluza-Klein modes associated to the non-harmonic chains, summarized in table 1, appear as massive scalar fields in the effective theory.

3.2.1 The complex structure sector

Let us first discuss the four-dimensional fields associated to the holomorphic three-form Ω. Recall that we demanded in (3.2) that our manifold $\mathcal{M}_6$ is still complex. This implies that the space of complex structure deformations will admit a similar structure as in the Calabi-Yau case. We introduce the periods $(X^K, \mathcal{F}_K)$ as

$$X^K = \int_{\mathcal{A}^K_-} \Omega\,, \qquad \mathcal{F}_K = \int_{\mathcal{B}^-_K} \Omega\,, \qquad K = 0, \ldots, b^3_- - 1\,, \tag{3.16}$$

where $(\mathcal{A}^K_-, \mathcal{B}^-_K)$ is a real symplectic basis of $H^-_3(\mathcal{M}_6)$. Ω and its periods depend holomorphically on $b^3_- - 1$ complex structure deformations z^k. By the local Torrelli Theorem the complex structure deformations can be mapped locally one to one to the projective space spanned by the periods X^K. In special coordinates this map is given by $z^k = X^k/X^0$. Note that all cycles in (3.16) have to be in the negative eigenspace of σ^* due to (3.10). In particular, if Ω is integrated over a positive cycle $\mathcal{A}_i + \mathcal{A}'_i$ one has

$$X^i_+(z) \equiv X^0 z^i_+ = \int_{\mathcal{A}_i + \mathcal{A}'_i} \Omega = 0\ . \tag{3.17}$$

This condition is in accord with (3.9) where it was imposed to ensure that $d\Omega = 0$. Similar constrains have to be imposed for integrals over elements in $H^+_3(\mathcal{M}_6)$. Note however, that there are U(1) vectors arising from the integrals of the R-R four-form C_4 over these positive cycles. In accord with (3.11) one has

$$A^i_{\mathrm{H}} = \int_{\mathcal{A}_i + \mathcal{A}'_i} C_4\,, \qquad A^\kappa = \int_{\mathcal{A}^+_\kappa} C_4\,, \qquad \kappa = 1, \ldots, b^3_+\,, \tag{3.18}$$

where $\mathcal{A}^+_\kappa$ is a basis of $H^+_3(\mathcal{M}_6)$. Note that the vectors arising from integrals over the symplectic dual cycles of $\mathcal{A}_i + \mathcal{A}'_i$ and $\mathcal{A}^+_\kappa$ do not contain new degrees of freedom due to the self-duality of C_4 as discussed in section 2.2. While in the underlying $\mathcal{N} = 2$ theory the complex structure deformations pair with the U(1) vectors into $\mathcal{N}=2$ vector multiplets the orientifold splits these into $\mathcal{N}=1$ chiral and vector multiplets. One thus finds b^3_- chiral multiplets with bosonic scalars z^k and $b^3_+ + N$ vector multiplets with bosonic components $A^i_{\mathrm{H}}, A^\kappa$.

The $\mathcal{N}=1$ characteristic functions for z^k and the vectors $A^i_{\mathrm{H}}, A^\kappa$ are readily determined following [28, 29]. The Kähler potential for the complex structure deformations z^k takes the well-known form

$$K_{\mathrm{cs}}(z,\bar{z}) = -\log\Big[-i\int \Omega\wedge\bar{\Omega}\Big]\,. \tag{3.19}$$

The gauge-kinetic coupling functions of the U(1) vectors $A^i_{\mathrm{H}}, A^\kappa$ will be denoted by $f^{\mathrm{H}}_{ij}, f_{\kappa\lambda}$ and $f_{\kappa i}$. They are holomorphic functions of the complex structure deformations z^k. In general there will be a kinetic mixing between the A^i_{H} and A^κ through $f_{\kappa i}$. For simplicity, we will assume that this mixing is small such that the massless A^κ decouple from the A^i_{H}.[3] Our main focus will be on the vectors A^i_{H} with gauge-coupling function given by

$$f^{\mathrm{H}}_{ij}(z) = -i\frac{\partial^2\mathcal{F}}{\partial z^i_+\partial z^j_+}\bigg|_{z^i_+=0}\,, \tag{3.20}$$

where $\mathcal{F}(z)$ is the $\mathcal{N}=2$ pre-potential depending on all complex structure deformations z^i_+, z^k defined after (3.16) and in (3.17). As explained in more detail in ref. [28], the $\mathcal{N}=1$ gauge-kinetic coupling function is obtained by first taking derivatives with respect to z^i_+ and then restricting to the orientifold locus $z^i_+=0$. We will argue in section 3.4 that K_{cs} and $f^{\mathrm{H}}_{ij}(z)$ should be calculable at least near the local hidden singularity.

3.2.2 The non-Kähler sector

Let us now turn to the $\mathcal{N}=1$ chiral multiples arising from J and B_2. It was shown in ref. [28, 29] that the orientifold theory enforces a particular complex structure on the $\mathcal{N}=1$ chiral field space which combines the NS-NS fields with the R-R fields into complex scalars. More explicitly, one introduces

$$\begin{aligned}\tau &= C_0 + ie^{-\phi}\,, \qquad G^a = \int_{\Gamma_a}(C_2-\tau B_2)\,,\\ T_M &= -\int_{\tilde{\Sigma}^M} e^{-B_2}\wedge C^{\mathrm{RR}} + ie^{-\phi}\int_{\tilde{\Sigma}^M}\frac{1}{2}\big(J\wedge J - B_2\wedge B_2\big)\,,\end{aligned} \tag{3.21}$$

where $C^{\mathrm{RR}} = C_0+C_2+C_4$. The two-cycles Γ_a are introduced in (3.15), while the four-chains $\tilde{\Sigma}^M = (\tilde{\Sigma}^\alpha, \tilde{\Sigma}^i)$ consist of a basis $\tilde{\Sigma}^\alpha$ of $H_4^+(\mathcal{M}_6)$ and the chains $\tilde{\Sigma}^i$ introduced in (3.13). Note that τ, G^a, T_M are nothing else then the integrals of $e^{-B_2}\big(C^{\mathrm{RR}} + ie^{-\phi}\,\mathrm{Re}(e^{iJ})\big)$ over one, two and four-chains respectively. These chains have to transform with definite signs under σ^* to due to the transformation (3.10) and (3.11) of J, B_2 and C_p. The Kähler

[3] Note that the presence of kinetic mixing can have an interesting effect on the visible phenomenology as discussed recently in a string theory context in ref. [33] (see also the references therein).

potential for τ, G^a, T_M is given by [29][4]

$$K_{\rm q} = -2\log\left[e^{-2\phi}\mathcal{V}\right]\,, \qquad \mathcal{V} = \frac{1}{3!}\int_{\mathcal{M}_6} J\wedge J\wedge J\,, \tag{3.22}$$

where $\mathcal{V}$ is the string-frame volume of the compact manifold $\mathcal{M}_6$. The total Kähler potential for the bulk modes is then given by $K = K_{\rm cs} + K_{\rm q}$, with $K_{\rm cs}$ as in (3.19).

In order to evaluate the Kähler metric, one needs to evaluate the Kähler potential (3.22) as a function of the $\mathcal{N}=1$ coordinates τ, G^a and T_M defined in (3.21). This is in general very hard, in particular since the internal manifold is not Kähler and we cannot apply $\mathcal{N}=2$ special geometry. However, the derivatives of $K_{\rm q}$ can be evaluated using the work of Hitchin [34] as done in [29]. Firstly, one notes that $K_{\rm q}$ only depends on the dilaton $e^{-\phi}$ and J. It was shown in [34] that $e^{-2\phi}\mathcal{V}$ is a well-defined functional of $e^{-\phi}{\rm Re}(e^{iJ})$ as long as this form is closed. This is indeed the case, since we imposed $dJ\wedge J = 0$ in (3.2) and we are in the orientifold limit where ϕ is constant on $\mathcal{M}_6$. This ensures that $K_{\rm q}$ can be evaluated as function of ${\rm Im}\,\tau$ and $e^{-\phi}\int_{\tilde\Sigma^M} J\wedge J$. In order to translate the latter into a dependence on ${\rm Im}\,T_M$ one needs to compensate the B_2 in ${\rm Im}\,T_M$ using ${\rm Im}\,G^a$. This can be done consistently on each divisor $\tilde\Sigma^M$ and in particular for the four-cycle S providing the visible sector. More generally, it was shown in ref. [34] that $e^{-2\phi}\mathcal{V}$ can be evaluated as a function of $e^{-\phi}{\rm Re}(e^{-B_2+iJ})$ as long as this form is closed under $d + H_3\wedge$, where $H_3 = \langle dB_2\rangle$ is the NS-NS three-form flux.

For the evaluation of the Kähler metric it is essential that the Kähler potential does not depend on the R-R forms C_0, C_2 and C_4. In particular, note that in contrast to $J\wedge J$ the four-form C_4 is not closed. This non-closedness results in a gauging of the scalars T_i since

$$\int_{\tilde\Sigma^i} d(e^{-B_2}\wedge C^{\rm RR}) = d_4{\rm Re}T_i + \int_{\tilde\Sigma^i} d_6 C_4 = d_4{\rm Re}T_i + e_{ij}A^i_{\rm H}\,, \tag{3.23}$$

where d_4 and d_6 are the differentials in the visible and compact dimensions respectively, and we have used (3.12) in evaluating the last equality. This implies that we have to replace the ordinary derivatives in the four-dimensional kinetic terms for $T_M = (T_\alpha, T_i)$ by the covariant derivatives

$$\mathcal{D}T_\alpha = dT_\alpha\,, \qquad \mathcal{D}T_i = dT_i + ie_{ij}A^j_{\rm H}\,, \tag{3.24}$$

where $A^i_{\rm H}$ are the U(1) vector fields in (3.18). The gauging of T_i will induce a D-term providing a potential for the modes arising from non-harmonic forms. Note that the $\mathcal{N}=2$ analog of the gauging (3.23) has been studied in refs. [23].[5]

3.2.3 The scalar potential

So far we discussed the kinetic terms for the vectors and scalars in the $\mathcal{N}=1$ effective action. Even in the absence of background fluxes, we expect that a potential is generated

[4]Strictly speaking this Kähler potential is valid only in the large volume limit of $\mathcal{M}_6$ and will receive corrections once cycles in $\mathcal{M}_6$ become small.

[5]In this case, the analysis of the kinetic terms is more complicated since the R-R fields reside in the quaternionic geometry and the application of Hitchins work is likely to be more involved.

for the compactification on $\mathcal{M}_6$. In particular it should give a mass to the fields arising from the non-Kähler deformation resolving the singular Calabi-Yau space. Such a potential will arise precisely from the gauging (3.24) of the scalars T_i. Recall that the $\mathcal{N}=1$ scalar potential is of the form

$$V = e^K \big(G^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \big) + \frac{1}{2} (\mathrm{Re} f)^{KL} D_K D_L \,, \tag{3.25}$$

where $(\mathrm{Re} f)^{KL}$ is the inverse of the real part of the gauge-coupling function f_{KL}. Focusing on (3.24) we note that e_{ij} is invertible and all N complex fields T_i are gauged. This implies that there are N D-terms D_i^{H} induced in the potential (3.25). In the case at hand we evaluate using (3.21), (3.22) and (3.12) that [6]

$$D_i^{\mathrm{H}} = -i e_{ij}\, \partial_{T_i} K = 4 e^{K_{\mathrm{q}}/2}\, e^{-\phi} \int_{\mathcal{B}^i + \mathcal{B}'^i} dJ \,, \tag{3.26}$$

where K_{q} is given in (3.22). The simple form of this D-term arises due to the fact that there are no other scalars in the spectrum charged under A^i_{H}. One might have suspected that, at least at the special locus where some of the three-cycles become very small, additional states charged under A^i_{H} arise. These would correspond to light D3-branes wrapped on the vanishing cycles and contribute light hypermultiplets in the underlying $\mathcal{N}=2$ theory [35]. However, such states are actually absent if there is a R-R flux on the shrinking three-cycle [36].

As expected for a non-Kähler reduction, the D-term (3.26) and the gauging (3.24) will induce a mass for the complex scalars T_i. However, it remains to discuss the scalar potential for the fields G^a defined in (3.21) as integrals of C_2 and B_2 over the negative cycles Γ_a. Such a mass term will arise from the the reduction of the ten-dimensional term

$$\frac{1}{4} \int_{10} e^{\phi}\, G_3 \wedge *_{10} \bar{G}_3 = \int_4 *_4 \mathbf{1}\ m_{ab}\, G^a \bar{G}^b + \ldots \,, \tag{3.27}$$

where $G_3 = F_3 - \tau H_3$ contains the field strengths of C_2 and B_2. The mass m_{ab} will depend on the size of the three-chains with boundary cycles Γ_a. The mass term (3.27) can also be translated into an $\mathcal{N}=1$ potential (3.25) and will arise from a superpotential. This W is of the form (2.18), i.e. given by

$$W = \int_{\mathcal{M}_6} G_3 \wedge \Omega \,, \tag{3.28}$$

where G_3 is the internal part of the field strengths of C_2, B_2. This W should contain terms linear in G^a arising form the exact forms $d_6(C_2 - \tau B_2)$, where d_6 is the differential on $\mathcal{M}_6$. However, in order to derive (3.27) from (3.28) one has to allow variations of Ω which are non-closed and hence leave the class of complex manifolds. The consideration of this extended class of spaces is crucial since $\mathcal{M}_6$ is compact. On the local non-compact Calabi-Yau geometries of $\mathcal{M}_6$ one cannot move d_6 onto Ω in (3.28). Therefore, the non-compact

[6] Recall the general formula for the D-term of a U(1) symmetry, $K_{I\bar{J}} \bar{X}^{\bar{J}}_k = \partial_I D_k$, where X^J is the Killing vector of the U(1) symmetry given by $\delta M^I = \Lambda^k X^J_k \partial_J M^I$.

case allows to model the non-complex geometries by changing the boundary conditions for the non-compact cycles (see the recent discussion in [37, 38]).

For the discussion of U(1) mediation the fields G^a will be of no importance. We will make use of the fact that they are massive and can be fixed to vevs where $\mathcal{M}_6$ is complex. The flux superpotential (3.28) can also fix the dilaton τ as well as the complex structure moduli. We will assume that the only light complex structure moduli arise from the hidden singularity discussed in section 3.4 and trigger supersymmetry breaking. Finally, in order to stabilize all moduli of the theory, one should also fix the moduli T_α supersymmetrically by, for example, using the mechanisms proposed in [39].

3.3 The visible sector on D-branes

In this section we discuss the inclusion of space-time filling D-branes which provide the visible sector in the effective Lagrangian (2.1). We apply and generalize the results of refs. [40 − 42] to the orientifold compactifications of section 3.2. The four-dimensional theory will admit charged chiral matter fields if this sector consists of a number of intersecting D-branes. In order to provide the ground for the examples considered in section 4, we will concentrate on branes at singularities of the internal space. The local geometry allows the D-branes to split up into several intersecting fractional branes which can be engineered to yield a semi-realistic visible spectrum [1 − 3].[7]

The singularities we will consider are obtained by shrinking a four-cycle in $\mathcal{M}_6$ to a point. In the following we will exemplify the general strategy for the case of del Pezzo surfaces. These are either the surfaces $\mathcal{B}_n$ which are obtained by blowing up $\mathbb{P}^2$ on n generic points, or $\mathbb{P}^1 \times \mathbb{P}^1$. The del Pezzo $\mathcal{B}_n$ has Hodge numbers $h^{(1,1)} = n + 1$ and $h^{(2,2)} = h^{(0,0)} = 1$, with all other numbers vanishing, so that the Euler number is

$$\chi(\mathcal{B}_n) = \int_{\mathcal{B}_n} c_2 = 3 + n \ . \tag{3.29}$$

Using Hirzebruch-Riemann-Roch [43] gives for the arithmetic genus of a surface S: $\chi_0 = \sum_{p=0}^{\dim(S)} (-1)^p h^{p,0} = \frac{1}{12} \int_S (c_1^2 + c_2)$, and since $\chi_0(\mathcal{B}_n) = h^{0,0} = 1$ we get

$$K^2 = \int_{\mathcal{B}_n} c_1^2 = 9 - n \ . \tag{3.30}$$

One calls K^2 also the degree of the del Pezzo surface. A base of homologically nontrivial two-cycles in $\mathcal{B}_n$ consists of the class of lines l in $\mathbb{P}^2$ as well as the n exceptional curves with classes e_i corresponding to the blow-ups. The intersection numbers are $l^2 = 1$ and $e_i^2 = -1$ with all other intersections vanishing.

As an alternative basis on can use the degree zero sublattice of $H_2(\mathcal{B}_n, \mathbb{Z})$ which has zero intersection with the canonical class

$$K = -3l + \sum_i e_i \ . \tag{3.31}$$

[7]For our purposes, it will not be crucial to model a fully realistic MSSM.

class	$\mathcal{B}_1$	$\mathcal{B}_2$	$\mathcal{B}_3$	$\mathcal{B}_4$	$\mathcal{B}_5$	$\mathcal{B}_6$	$\mathcal{B}_7$	$\mathcal{B}_8$
$(0;-1)$	1	2	3	4	5	6	7	8
$(1;1^2)$		1	3	6	10	15	21	28
$(2;1^5)$					1	6	21	56
$(3;2,1^6)$							7	56
$(4;2^3,1^5)$								56
$(5;2^6,1^2)$								28
$(4;3,2^7)$								8
Total no.	1	3	6	10	16	27	56	240

Table 2: Number of lines on the $\mathcal{B}_n$ del Pezzo surfaces. The coefficients $(a; b_1, \ldots b_n)$ describing the classes are given w.r.t. the generators $(l; -e_1, \ldots, -e_n)$.

For $n \geq 3$ this lattice is identified with the root lattice of the groups $\mathcal{E}_n$, where for $n = 6, 7, 8$ $\mathcal{E}_n = E_n$ are the exceptional groups E_n, $\mathcal{E}_5 = D_5$, $\mathcal{E}_4 = A_4$ and $\mathcal{E}_3 = A_2 \oplus A_1$. Defining the simple roots by

$$\alpha_i = e_i - e_{i+1} \,, \quad i = 1, \ldots, n-1 \,, \qquad\qquad \alpha_n = l - e_1 - e_2 - e_3 \tag{3.32}$$

it is immediate that the intersection matrix of α_i is given by the negative Cartan matrix $-C^{ij}$ of the corresponding Lie algebra. The intersection matrix $\mathcal{K}^{IJ}$ of the cycles $\Sigma_I = (K, \alpha_i)$, $I = 0, \ldots, n$ is thus given by

$$\mathcal{K}^{00} = 9 - n \,, \qquad \mathcal{K}^{ij} = -C^{ij} \,, \qquad \mathcal{K}^{0i} = 0 \;. \tag{3.33}$$

Let C be a curve in the del Pezzo surface. Then its degree $deg(C)$ and its arithmetic genus g reads

$$deg(C) = -K.C \,, \qquad\qquad g = \frac{1}{2}(C.C + K.C) + 1 \;. \tag{3.34}$$

Lines on the del Pezzo surface fall into representations of the Weyl-Group. To understand the geometry of the embedding of del Pezzo surfaces in Calabi-Yau spaces it is useful that the number of some lines will appear as rational instantons of degree one in the classes realized globally in the embedding Calabi-Yau. For convenience of the reader we reproduce table 3 of ref. [44].

In the del Pezzo surface $\mathcal{B}_n$ there are $n+1$ Kähler parameters associated with the volumes of the two-cycles (K, α_i). In general, for $\mathcal{B}_n$ in some compact Calabi-Yau space $\iota : \mathcal{B}_n \hookrightarrow Y$, not all of the two-cycles will descend to Kähler moduli of Y. The number of associated Kähler moduli is determined by the rank of the map

$$\Pi : \; H_2(\mathcal{B}_n) \;\to\; H_2(Y) \,, \qquad\qquad \Pi(\Sigma, \omega) = \int_\Sigma \iota^* \omega \,, \tag{3.35}$$

where $\Sigma \in H_2(\mathcal{B}_n)$ and $\omega \in H^2(Y)$. Using the orientifold involution σ we can split $H_2(\mathcal{B}_n)$ into eigenspaces $H_2^+(\mathcal{B}_n) \oplus H_2^-(\mathcal{B}_n)$ with basis $(\Sigma_\kappa^+, \Sigma_a^-)$ and accordingly decompose the intersections (3.33) as

$$\mathcal{K}_+^{\kappa\lambda} = \Sigma_\kappa^+ \cdot \Sigma_\lambda^+ \,, \qquad \mathcal{K}_-^{ab} = \Sigma_a^- \cdot \Sigma_a^- \,, \tag{3.36}$$

with the mixed intersections $\Sigma^+_\kappa \cdot \Sigma^-_a$ vanishing. Since ι commutes with σ, one can split Π into maps from the σ-eigenspaces $H^\pm_2(\mathcal{B}_n)$ to $H^\pm_2(Y)$.

Note that in the case that the internal manifold is a non-Kähler space $\mathcal{M}_6$ as in section 3.2 the map Π will no longer identify cohomology elements of $\mathcal{M}_6$ and $\mathcal{B}_n$, but should also include the boundary cycles Σ_i and Γ_a of table 1. Correspondingly, the new map can be of higher rank. Following our considerations of section 3.2 the cycles Σ_i, Γ_a will precisely be in the four-cycle $S = \mathcal{B}_n$ supporting the visible sector, i.e.

$$\Sigma_i \in H^+_2(\mathcal{B}_n)\,, \qquad \Gamma_a \in H^-_2(\mathcal{B}_n)\ . \tag{3.37}$$

In summary, there will be two types of cycles: two-cycles in $H_2(\mathcal{B}_n)$ which are non-trivial in $H_2(\mathcal{M}_6)$, as well as two-cycles which are homologically trivial in $\mathcal{M}_6$. Recall that we are taking $b^2_- = 0$, such that all elements of $H^-_2(\mathcal{B}_n)$ are trivial in $\mathcal{M}_6$. In section 4.3 we will discuss how one can count the number of cycles of the various types, and also show that on a del Pezzo $H^2_-(\mathcal{B}_n)$ is always non-empty if σ acts non-trivially on $\mathcal{B}_n$.

Let us now wrap branes on the surface $\mathcal{B}_n$. The non-trivial zero-, two- and four-cycles in $\mathcal{B}_n$ can support fractional space-time filling D-branes. We will denote the field strength of the k^{th} stack of fractional branes by $\mathcal{F}^k$. The number of $D3$, $D5$ and $D7$ branes are encoded by the charge vector

$$\text{ch}(\mathcal{F}^k)\ \cong\ (r_k, p^I_k, q_k)\ . \tag{3.38}$$

The $D7$ charge is the rank r_k of $\mathcal{F}^k$, the D5 charge is captured by fluxes $p^I_k \cong (p^\kappa_k, p^a_k)$ on two-cycles $\Sigma_I \cong (\Sigma^+_\kappa, \Sigma^-_a)$ in $\mathcal{B}_n$, and the D3 charge is encoded by the instanton number q_k. Explicitly, these are given by

$$p^\kappa_k = \int_{\Sigma^+_\kappa} \text{Tr}(\mathcal{F}^k)\,, \qquad p^a_k = \int_{\Sigma^-_a} \text{Tr}(\mathcal{F}^k)\,, \qquad q_k = \frac{1}{2}\int_{\mathcal{B}_n} \text{Tr}(\mathcal{F}^k \wedge \mathcal{F}^k)\ . \tag{3.39}$$

This form of the charge vector can be inferred from the coupling of the k^{th} brane stack to the R-R forms C^{RR} via the Chern-Simons action[8]

$$S^{\text{CS}} = \int_{\mathbb{M}_{3,1}\times\mathcal{B}_n} \iota^* C^{\text{RR}} \wedge e^{\mathcal{F}^k - \iota^* B_2}\ . \tag{3.40}$$

where $C^{\text{RR}} = C_0 + C_2 + C_4$ is the sum of the R-R potentials as in (3.21). It turns out to be convenient to also define the topological numbers $q_{k\,\kappa} = p^\lambda_k \mathcal{K}^+_{\lambda\kappa}$ and $q_{k\,a} = p^b_k \mathcal{K}^-_{ba}$, where we lower the indices with the inverses of the intersection matrices given in (3.36).

Expanding the Chern-Simons action and integrating over the four-cycle $\mathcal{B}_n$ one encounters the term of the form $\text{Im} f^{\text{V}}_k\, F^k \wedge F^k$, which contains the four-dimensional part F^k of the field strength $\mathcal{F}^k$. This term determines the imaginary part of the gauge-coupling function f^{V}_k. Using the fact that f^{V}_k is holomorphic in the $\mathcal{N} = 1$ coordinates defined in section 3.2, it is determined to be

$$f^{\text{V}}_k = -i(T_{\mathcal{B}_n} - q_{k\,a} G^a - q_k\,\tau)\,, \tag{3.41}$$

[8]Note that in general there will be corrections to S^{CS} given by $\sqrt{\hat{A}_T/\hat{A}_N}$, where $\hat{A}_T, \hat{A}_N$ are the $\hat{A}$ genera of the tangent and normal bundle to the brane world-volume. These will equivalently appear in the definition (3.21) of the $\mathcal{N} = 1$ coordinates.

where $T_{\mathcal{B}_n}, G^a, \tau$ are the $\mathcal{N}=1$ coordinates defined in (3.21). In particular, $T_{\mathcal{B}_n}$ is the Kähler structure coordinate (3.21) corresponding to the del Pezzo four-cycle $\mathcal{B}_n$ and takes the form

$$T_{\mathcal{B}_n} = -\rho_{\mathcal{B}_n} + ie^{-\phi}\,\mathrm{vol}_{\mathcal{B}_n} - \frac{1}{2(\tau-\bar{\tau})}\mathcal{K}^-_{ab} G^a (G-\bar{G})^b\,, \tag{3.42}$$

where $\rho_{\mathcal{B}_n}$ is the R-R axion and $\mathrm{vol}_{\mathcal{B}_n}$ is the volume of $\mathcal{B}_n$.

In addition to the gauge-coupling function, there are also further couplings of the R-R four-form to the gauge-theory. The expansion of the Chern-Simons action (3.40) yields the term

$$\int_{\mathcal{B}_n} \mathrm{Tr}(\mathcal{F}^k \wedge \mathcal{F}^k) \wedge \iota^* C_4 = q_{k\,M}\ F^k \wedge \mathcal{C}^M + \dots\,, \tag{3.43}$$

where $\mathcal{C}^M = (\mathcal{C}^i, \mathcal{C}^\alpha)$ are the four-dimensional two-forms dual to the R-R four-form scalars in $\mathrm{Re}\,T_M$ defined in (3.21). Clearly, $q_{k\,A}$ vanishes if $\tilde{\Sigma}^M$ in (3.21) has no intersection with the two-cycle in $\mathcal{B}_n$ supporting the $D5$ charge. The contribution (3.43) to the effective action is of the form (2.9) and hence induces a gauging of the scalars $\mathrm{Re}\,T_M$. This implies that the covariant derivatives (3.24) are modified to

$$\begin{aligned} DT_\alpha &= dT_\alpha + iq_{k\,\alpha} A^k_{\mathrm{V}}\,, \\ DT_i &= dT_i + i(e_{ij}\, A^j_{\mathrm{H}} + q_{k\,i}\, A^k_{\mathrm{V}})\,, \end{aligned} \tag{3.44}$$

where A^k_{V} is the visible $\mathrm{U}(1)_k$ factor on the k^{th} D-brane stack. Note that the fields T_i can thus be gauged by both the hidden and visible sector gauge-fields. As in eq. (2.11), we can now identify heavy and light mass eigenstates proportional to $e_{ij}\, A^j_{\mathrm{H}} + q_{a\,i}\, A^a_{\mathrm{V}}$ $e_{ij}\, A^j_{\mathrm{H}} - q_{a\,i}\, A^a_{\mathrm{V}}$. As explained in section 2.1, the linear combinations appearing in (3.44) can become heavy via the Higgs mechanism and can be integrated out. The remaining light vector fields then couple to both the hidden flux geometry as well as the visible gauge theory on the fractional branes and can mediate supersymmetry breaking.

We are now in the position to derive the D-term potential for the whole configuration. In addition to the bulk D-term (3.26) we have additional contributions

$$D^{\mathrm{V}}_a = 4e^{K_{\mathrm{q}}} \int_{\mathcal{B}_n} \mathrm{Tr}(\mathcal{F}_a) \wedge \iota^* J - \sum_i Q^{(a)}_i |\phi_i|^2\,, \tag{3.45}$$

where $Q^{(a)}_i$ are the $\mathrm{U}(1)_a$ charges of the canonically normalized matter fields ϕ_i. The full D-term potential is now obtained by inserting (3.20), (3.26) as well as (3.41) and (3.45) into (3.25).

3.4 The hidden flux geometry and supersymmetry breaking

In this section we study the hidden sector supporting a supersymmetry breaking flux background. Supersymmetry breaking by background fluxes has been investigated since the advent of flux compactifications [15]. It was shown that warped Calabi-Yau compactifications with flux superpotential (2.18) admit supersymmetric vacua at points in the moduli space where the complex flux G_3 is a $(2,1)$ form. If this condition is violated supersymmetry appears to be broken spontaneously. Unfortunately, in the full compactification such a conclusion can be too quick. The effects of the supersymmetry breaking fluxes have to be sufficiently small and localized to ensure persisting control over the effective four-dimensional theory and moduli stabilization. Moreover, supersymmetry breaking fluxes can backreact on the internal geometry and render the compact manifold to be no longer Calabi-Yau.

A simple way to obtain meta-stable non-supersymmetric flux vacua in *local* Calabi-Yau geometries has been studied in ref. [19]. We will use orientifolds of the set-ups [19] to model a supersymmetry breaking sector. In order to do that, we zoom into local regions $\hat{X}, \hat{X}'$ of the compactification manifold $\mathcal{M}_6$. Let us assume that $\hat{X}$ and $\hat{X}'$ are locally Calabi-Yau and get exchanged under the geometric orientifold involution σ. In order to not worry about cross couplings connecting fields on $\hat{X}$ and its orientifold image $\hat{X}'$ we will demand that these regions are away from the orientifold planes in $\mathcal{M}_6$. This will allow us to work on $\hat{X}$ keeping in mind that there exists an identical copy $\hat{X}'$.

Considering type II string theory on the non-compact Calabi-Yau $\hat{X}$ forces us to decouple gravity. Nevertheless, we can study the moduli space of complex structure deformations $\underline{S} = (S^i)$, $i = 1, \ldots, h^{2,1}(\hat{X})$ by analyzing the variations of the holomorphic three-form $\Omega(S)$. Let us focus on the cases where $\hat{X}$ contains a number of compact three-cycles $\mathcal{A}_i$ with S^3 topology. A simple example $\hat{X}$ is obtain as deformations of a fibered A_1 singularity [45]. Such a local geometry is given by a complex equation in $\mathbb{C}^4$ of the form

$$u^2 + w^2 + v^2 + p_m(t)^2 + f_{m-1}(t|\underline{S}) = 0\,, \qquad p_m = g\prod_i (t - a_i)\,, \tag{3.46}$$

where the subscripts indicate the degree of the polynomial functions in t. The local geometry (3.46) can be described as follows. If $f_{m-1}(t|\underline{S}) = 0$ one obtains m nodal singularities of the local form (4.4) at $u = w = v = 0$ and the roots a_i, $i = 1, \ldots, m$ of $p_m(t)$. The $f_{m-1}(t|\underline{S})$ destroys the factorization in $t - a_i$ and deforms the nodes into m S^3's denoted by $\mathcal{A}_i$. These S^3's are homologically distinct and their size can be parameterized by m independent complex structure deformations $\underline{S}$. In order to make contact to the discussion of section 3.2 we note that $\mathcal{A}_i$ should include the cycles $\mathcal{A}_i$ introduced in (3.13). However, in full analogy to the discussion of the visible sector in section 3.3, the relations (3.12) only arise through the embedding of $\hat{X}$ and $\hat{X}'$ into the global non-Kähler space $\mathcal{M}_6$.

The fact that $\hat{X}$ and its orientifold image are Calabi-Yau allows us to use $\mathcal{N} = 2$ special geometry to describe the moduli space spanned by $\underline{S}$. One introduces the non-compact cycles $\mathcal{B}^i$ which are the symplectic duals to the compact three-cycles $\mathcal{A}_i$ in $\hat{X}$. This is possible if one introduces a cutoff Λ_0 to regulate integrals over $\hat{X}$. The periods of Ω are thus given by

$$S^i = \int_{\mathcal{A}_i} \Omega\,, \qquad \partial_i \mathcal{F} = \int_{\mathcal{B}^i}^{\Lambda_0} \Omega\,, \tag{3.47}$$

where $\partial_i \mathcal{F} = \partial_{S^i} \mathcal{F}$, and special geometry ensures the existence of a holomorphic pre-potential $\mathcal{F}(\underline{S})$. For the fibered A_1 singularity (3.46) the B-periods $\partial_i \mathcal{F}$ have been computed in ref. [45]. At leading order they take the simple form

$$2\pi i \partial_i \mathcal{F} = S^i \left[\log\left(\frac{S^i}{p'_m(a_i)\Lambda_0^2} \right) - 1 \right] + \sum_{i \neq j} S^j \log\left(\frac{\Delta_{ij}^2}{\Lambda_0^2} \right) + \ldots\,. \tag{3.48}$$

Here $p'_m(a_i)$ is the first derivative of p_m introduced in (3.46) evaluated at the root a_i, and $\Delta_{ij} = a_i - a_j$ is the complex distance between the nodes in the singular $\hat{X}$.

In the $\mathcal{N}=2$ theory the complex scalars S^i sit together with U(1) vectors $A^i = \int_{\mathcal{A}_i} C_4$ in vector multiplets. Note that on the orientifold image $\hat{X}'$ we can similarly introduce the periods $(S'^i, \partial'_i \mathcal{F})$ on the image cycles $(\mathcal{A}'_i, \mathcal{B}'^i)$. The orientifold projection (3.10), (3.11) ensures that the $\mathcal{N}=2$ vector multiplets (S^i, A^i) and (S'^i, A'^i) split into $\mathcal{N}=1$ chiral multiplets with complex scalars $s^i = (S^i - S'^i)/2$ and $\mathcal{N}=1$ vector multiplets with $A^i_{\mathrm{H}} = A^i + A'^i$ $(A^i - A'^i = 0)$. As in (3.17) the orientifold locus is given by $S^i_+ = (S^i + S'^i)/2 = 0$, while the condition $\partial\mathcal{F}/\partial S^i_+ = 0$ automatically arises on the orientifold locus due to the symmetries of $\mathcal{F}$. The orientifolded four-dimensional effective theory will be a rigid $\mathcal{N}=1$ supersymmetric theory.

Restricting the theory to the orientifold moduli space parameterized by $\underline{s}$, the $\mathcal{N}=1$ metric remains rigid special Kähler with Kähler potential

$$K(\underline{s}, \bar{\underline{s}}) = \frac{i}{2}(s^j \partial_{\bar{s}^j} \bar{\mathcal{F}} - \bar{s}^j \partial_{s^j} \mathcal{F})_{\underline{S}_+=0} \, . \tag{3.49}$$

The Kähler metric is simply given by $K_{i\bar{j}} = \mathrm{Im}(\partial^2\mathcal{F}/\partial s^i \partial s^j)$ restricted to $\underline{S}_+ = 0$. The holomorphic function $f^{\mathrm{H}}_{ij}(\underline{s}) = -i\partial^2\mathcal{F}/(\partial S^i_+ \partial S^j_+)$ for $\underline{S}_+ = 0$ is the gauge-kinetic coupling function of the U(1) vectors A^i_{H} as in (3.20). In order to allow for U(1) mediation the embedding of $\hat{X}$ and $\hat{X}'$ into the compact space $\mathcal{M}_6$ has to ensure that A^i_{H} combines with a visible U(1) vector A^i_{V} into a light and massive eigenstate A^i, A^i_{h} as in (2.11). Upon integrating out A^K_{h} as in section 2, only A^K remains in the low energy theory and has an effective gauge-coupling f_{KL} given at lowest order the sum of the visible and hidden f's as in (2.12).

Let us turn to the scalar potential for $\underline{s}$. In rigid $\mathcal{N}=1$ supersymmetry it takes the form

$$V = K^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W} + \frac{1}{2}(\mathrm{Re} f)^{-1\, ij} D_i D_j \, , \tag{3.50}$$

where $\partial_i W = \partial_{s^i} W$, and D_i is the D-term for the light U(1) vector A^i. The superpotential W arises due to a non-trivial flux background and using (2.18) is given by

$$W = 2(\alpha_i\, s^i + N^i\, \partial_{s^i}\mathcal{F})_{\underline{S}_+=0} \, , \tag{3.51}$$

where the factor 2 arises due to the fact that there is an orientifold image of $\hat{X}$ in $\hat{X}'$. The flux quanta appearing in (3.51) are given by $N^i = \int_{\mathcal{A}_i} F_3$ and $\alpha^i = -\int_{\mathcal{B}^i}(F_3 - \tau H_3)$, where F_3 and H_3 are R-R and NS-NS three-form fluxes. Here τ is complex dilaton-axion (3.21) which, in the compact set-up, can be stabilized supersymmetrically by other background fluxes [46]. As in section 2 the leading contribution to the gaugino masses $\tilde{M}^{ij}$ of the light U(1) vector multiplets is given by the generalization of (2.4) with (2.12). Therefore, if there exists a non-supersymmetric minimum of the potential (3.50) the F-term F^m will be non-vanishing and contribute to $\tilde{M}^{ij}$.

A few comments concerning the presented outset are in order. Firstly, note that the described flux background was argued to be large-N dual to a set-up where the S^3's are replaced by $\mathbb{P}^1$'s via geometric transition. The fluxes N_i correspond to the rank of the gauge-group of N_i $D5$ or anti-D5 branes wrapped on the ith $\mathbb{P}^1$. The gauge-coupling function of the branes on the large-N dual of $\hat{X}$ at the scale Λ_0 is given by $\alpha(\Lambda_0) = \alpha_i$ for all stacks of branes. Secondly, note that the orientifold projection maps D5 to anti-D5 branes with the identification $N^i = -N'^i$ and $\alpha_i = -\alpha'_i$, since the fluxes F_3 and H_3 are odd under the orientifold involution. This implies that if the gauge-theory obtained from $\hat{X}$ arises from $D5$ branes only, the gauge-theory from $\hat{X}'$ is due to anti-D5 branes. The five-branes on $\hat{X}'$ have to wrap flopped $\mathbb{P}^1$'s similar to the recent discussion in ref. [37].

It was shown in refs. [19], that on geometries (3.46) one can indeed find meta-stable supersymmetry breaking flux vacua of (3.50). In order to simplify the discussion we will consider the cases $m = 1$ in the fibered A_1 singularity (3.46), such that each of the local Calabi-Yau spaces $\hat{X}$ and $\hat{X}'$ only admits one S^3's respectively. We will determine the minima of the scalar potential (3.50) in the situation of small D-terms. In order to do that we have to specify the sign of the fluxes N and Imα. If both have the same sign, there is a supersymmetric vacuum at $\langle s\rangle = g\Lambda_0^2\, e^{-2\pi i\alpha/N}$. Clearly, the gaugino mass $\tilde{M}$ vanishes in this vacuum. The situation changes as soon as one has opposite signs of N and Imα. In the case $N < 0$ and Im$\alpha > 0$ one finds a non-supersymmetric minimum of V at

$$\langle s\rangle = g\Lambda_0^2\, e^{2\pi i\bar{\alpha}/|N|}\,, \qquad \partial_s W = \alpha - \bar{\alpha}\ . \tag{3.52}$$

In this non-supersymmetric vacuum the gaugino mass $\tilde{M}$ is non-zero and evaluated to be $\tilde{M} \propto \tilde{g}^2|N|/\langle s\rangle$, where $\tilde{g}$ is the gauge-coupling of the mediating U(1). Note that even though this computation is very explicit and can be performed including higher corrections to the pre-potential it typically does not lead to the right scales for $\tilde{M}$ and supersymmetry breaking. This can be traced back to the fact that in the compact settings the fluxes are actually quantized in units of α'. Since the orientifold already specifies an $\mathcal{N} = 1$ supersymmetry inside the underlying $\mathcal{N} = 2$ theory, a small breaking cannot occur as in [19].

One expects that the scales can be made phenomenologically viable by placing the hidden singularity in a warped throat [47, 46, 48 − 50]. This implies that the ten-dimensional metric background is of the form $ds^2 = e^{2A}ds_4^2 + e^{-2A}ds_6^2$, where $ds_6^2(y)$ is the line element on $\mathcal{M}_6$, and the warp factor $e^{2A}(y)$ is depending on the internal coordinates. It is straightforward to verify that the warp factor cancels for the four-dimensional gauge-coupling function f^{H}. It is also believed that the warp factor does not induce leading corrections to the flux superpotential. However, there will be corrections to the $\mathcal{N} = 1$ Kähler potential. So far only the leading corrections have been analyzed in ref. [49], $\delta K \propto |s|^{2/3}$. These dominate for small $|s|$ such that the mass $\tilde{M}$ is of the form $\tilde{M} \propto \tilde{g}^2\langle|s|^{1/3}\partial_s W\rangle$. $\tilde{M}$ can be small if one finds non-supersymmetric vacua for sufficiently small $|s|$.

Let us end with a brief comment on an alternative to the route taken here. An interesting possibility is to model fluxed supersymmetry breaking in the hidden sector by using the supergravity backgrounds perturbed by anti-D3 branes in a warped throat [51, 52]. The anti-D3 brane will induce a supersymmetry breaking flux as required to generate the gaugino mass $\tilde{M}$. It would be interesting to analyze such set-ups on the level of the effective action and to study the resulting pattern of soft supersymmetry breaking terms. If supersymmetry breaking arises in a warped throat, partial sequestering can take place and one expects a mixing of gravity and gauge-mediated contributions to generate the visible soft masses.

4. Geometric realizations

In this section we discuss the geometric tools to construct internal manifolds with a hidden and visible sector. These sectors are realized near singularities of Calabi-Yau manifolds which become non-Kähler by the mechanism discussed in section 3.1. We start with a warm-up by recalling some basics on nodal singularities in section 4.1. For the hidden supersymmetry breaking sector we consider singularities with a number of deformed S^3's in section 4.2. For the visible sector we study the geometry of del Pezzo surfaces and realize them in simple compact Calabi-Yau spaces in section 4.3 and 4.4. The more involved compact examples admitting both the hidden as well as a visible del Pezzo singularity are discussed in section 5 and appendix A.

4.1 Simple nodal singularities

As a warm-up for the more involved singularities needed for the visible and hidden sector, we will first recall some basic facts about complete intersections and briefly discuss the simple example of the deformed or resolved conifold in quintic Calabi-Yau.

Most known Calabi-Yau spaces Y are given in terms of generically smooth embeddings

$$P_i(\underline{x}, \underline{z}) = 0\,, \qquad i = 1, \ldots, r\,, \tag{4.1}$$

into a toric ambient space T_Δ. The case $r = 1$ corresponds to the special case of a hypersurfaces, while $r > 1$ defines complete intersections. In (4.1) the $\underline{x}$ are the coordinates of T_Δ and $\underline{z}$ are complex deformation parameters of the polynomials. Generically smooth means that (4.1) and

$$dP_1 \wedge \ldots \wedge dP_r = 0\,, \tag{4.2}$$

have no solutions for generic values of the deformation parameters $\underline{z}$ and any value of $\underline{x}$. More precisely, the tangent space of complex structure deformations $H^1_{\bar\partial}(TY)$ is given in this situation generically by the space of infinitesimal deformations $\mathrm{def}(\underline{P})$ modulo the infinitesimal automorphisms $\mathrm{aut}(T_\Delta, \underline{P})$ of the ambient space, which are compatible with (4.1). The infinitesimal complex structure deformations can be extended to a global moduli space $\mathcal{M}^{\rm cs}$ without obstruction.[9] As in section 3.2 and with slight abuse of notation, we call the complex structure deformations $\underline{z}$. Note that solutions to (4.2) will generically exist in subloci of complex codimension one (or higher) in $\mathcal{M}^{\rm cs}$. At a codimension one locus the Calabi-Yau manifold Y can acquire a singularity for special values of $\underline{x}$. The most generic singularity is a node and it will be instructive to discuss some of the basic concepts for this simple case.

[9] The realization as complete intersection in particular toric ambient spaces can obstruct sometimes elements in $H^1_{\bar\partial}(TM)$. For simplicity we focus on complex structure deformations in $H^1_{\bar\partial}(TM)$ that can be realized by $\mathrm{def}(\underline{P})/\mathrm{aut}(T_\Delta, \underline{P})$.

JHEP10(2008)077

For the simplest realization of these concepts consider the famous quintic surface in the toric ambient space $\mathbb{P}^4$. The quintic surface has 101 complex structure deformations $\underline{z}$.[10] For simplicity consider the smooth one parameter family of quintics $P = \sum_{i=1}^5 x_i^5 - 5z\prod_{i=1}^5 x_i = 0$. Here $z \in \mathbb{C}$ is an unobstructed complex structure deformation. Generically the constraints $dP = 0$ and $P = 0$ have no solution, but at $z = 1$ and $x_i = 1$, $i = 1, \ldots, 5$ they have a solution, the conifold point. The conifold divisor $z = 1$ is codimension one in the one parameter family of quintic surfaces. We find that the local singularity is a node by expanding the defining equation of the Calabi-Yau manifold $P = 0$ in an affine chart $x_5 = 1$ for small $\mu = 1 - z$ near the singularity, i.e. at $x_i = 1 + \tilde{u}_i$, $i = 1, \ldots, 4$ with small $\tilde{u}_i$. After a linear change of variables from $\tilde{u}_i$ to (u, v, w, t), we can bring the local equation to the normal form

$$f = u^2 + v^2 + w^2 + t^2 - \mu(S) = 0\,, \tag{4.3}$$

The node occurs at $\mu(S) = 0$, where the real three sphere S^3 given by the real equation (4.3) is contracted to zero size. Switching on $\mu(S)$ deforms the node into an S^3, which can be parameterized by a complex structure deformation S. Locally, S is given by the special coordinate $S = \int_{S^3} \Omega(\mu)$. In the following we will use the letter S to denote the complex structure deformations in the local non-compact geometries such as (4.3).

By a further transformation the node singularity (4.3) at $\mu = 0$ can be brought in the form

$$\phi_1\phi_2 - \phi_3\phi_4 = 0\,, \tag{4.4}$$

and admits two kinds of small resolutions by an $\mathbb{P}^1$. Introducing two new projective complex coordinates (x, y) the smooth blown up geometry can be either described by (4.4) and the equations

$$\begin{pmatrix} \phi_1 & \phi_3 \\ \phi_4 & \phi_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \qquad \text{or} \qquad \begin{pmatrix} \phi_1 & \phi_4 \\ \phi_3 & \phi_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0\,. \tag{4.5}$$

At the point P given by $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$ the coordinates (x, y) describe a $\mathbb{P}^1$, while outside the singular point P (x, y) can be eliminated to recover the geometry (4.4). The holomorphic map

$$\pi : \quad \hat{X} \;\rightarrow\; X\,, \tag{4.6}$$

from the smooth resolution $\hat{X}$ to the nodal variety X is called the resolution map. It identifies $\hat{X}$ with X outside P. The resolution is called small, because the exceptional set $\pi^{-1}(P) = \mathbb{P}^1$ is of complex codimension two in the threefold. This implies that it does not affect the canonical class. The modification is local and holomorphic and does not affect the complex structure. As we discussed in section 3.1 the resolution is generically non-Kähler. We have recalled in section 3.1, that even if there are δ homology relations (3.3) among k shrinking three-cycles, there will also be many non-Kähler resolutions among the 2^k resolutions defined by (4.5) if $k \leq \delta$ [25].

[10] Generically a quintic constraint will have 126 deformation parameters. The dimension of $\mathrm{Aut}(\mathbb{P}^4, P) = \mathbb{C}^* \times PGL(5, \mathbb{C})$ is 25 rendering the number of complex parameters to 101.

Lie Algebra $\mathfrak{g}$	Polynom $g(w,v)$	Dual coxeter no. h
A_r	w^2+v^{r+1}	$r+1$
D_r	$w(v^2+w^{r-2})$	$2r-2$
E_6	w^3+v^4	12
E_7	$w(w^2+v^3)$	18
E_8	w^3+v^5	30

Table 3: A-D-E singularities.

4.2 The hidden singularity

In order to model the supersymmetry breaking hidden sector we have to built more complicated singularities. This can be achieved by going to higher codimension in the moduli space of a Calabi-Yau family with a large number of complex structure deformations. A classification of the local Calabi-Yau 3-fold singularities, which allow a resolution with trivial canonical bundle, has only been achieved in the hypersurface case [53].

4.2.1 Creating an A-D-E singularity

The most relevant examples for the hidden sector for us are non-compact Calabi-Yau manifolds with singularities, which admit small S^2 resolutions or S^3 deformations. They are of A-D-E type and given by the equation

$$u^2+g(w,v)+t^{mh}=0\,, \tag{4.7}$$

where $g(w,v)$ and the dual coxeter number h of the associated Lie algebra $\mathfrak{g}$ are listed in table 3. The requirement for the existence of a small resolution is $m\in\mathbb{N}$. Of course, for (4.7) embedded into a compact Calabi-Yau manifold there will be an upper bound on the rank of the (gauge) group associated to $\mathfrak{g}$ that is realizable.

In the following we will study orientifold involutions on geometries with A_r singularities for $r>1$. These are obtained by fibering the A_r two-fold in table 3 over a plane $\mathbb{C}[t]$ parameterized by t. The local equation of the Calabi-Yau space is now of the form [54]

$$P=uw-\prod_{i=1}^{r+1}(v-v^i(t))+f(v,t|\underline{S})=0\,,\qquad \sum_{i=1}^{r+1}v^i(t)=0\,, \tag{4.8}$$

where $v^i(t)$ are polynomials in t and $f(v,t|\underline{S})$ is a polynomial in v,t. The singular geometry is obtained by setting $f(v,t|\underline{S})=0$ and admits nodes located at $u=w=0$ and $v=v^i(t)=v^j(t)$ for $i\neq j$. If the highest power in t in this singular Calabi-Yau space is of the form (4.7) one finds $mr(r+1)/2$ nodes. The function $f(v,t|\underline{S})$ encodes the normalizable deformations of the singular Calabi-Yau space and is of the form [54]

$$f(v,t)=f_{m-1}(t)v^{r-1}+f_{2m-1}(t)v^{r-2}+\ldots+f_{(r-1)m-1}(t)v+f_{rm-1}(t)\;. \tag{4.9}$$

The coefficients in f_i correspond to the $mr(r+1)/2$ complex structure deformations $\underline{S}$ which deform the nodes into S^3's. It is easy to see that the general form (4.8) reduces to (3.46) for the A_1 singularity. In this case one has $v^1(t) = -v^2(t) = p_m(t)$ and only the first and last term $f_{m-1}(t)$ remain in (4.9).

4.2.2 Orientifolds of A-type singularities

We are now in the position to specify a local orientifold action on the A_r geometries (4.8) for $r = 2k$. We first write (4.8) as

$$uw - (v - v^0(t)) \prod_{i=1}^{k} (v - v^i(t)) \cdot \prod_{j=1}^{k} (v - \tilde{v}^j(t)) + f(v, t|\underline{S}) = 0 \; . \tag{4.10}$$

Let us briefly discuss two holomorphic orientifold symmetries of (4.10). The simplest case is a σ_1 which inverts one coordinate $t \mapsto -t$, while keeping all other directions invariant. This involution preserves an $O7$ plane at $t = 0$, and we need to demand that it exchanges v^i with $\tilde{v}^i$ and preserves v^0, f. A second possibility σ_2 is to map $(t, v, u, w) \mapsto (-t, -v, -w, u)$, such that (4.10) together with v^0, f are anti-invariant and $v^i \leftrightarrow \tilde{v}^i$. This involution has an $O3$ plane at $t = v = u = w = 0$.

The transformation properties of $f(v, t|\underline{S})$ under σ_1 or σ_2 will restrict the number of allowed complex structure deformations $\underline{S}$. An orientifold invariant deformation in (4.9) arises from monomials $t^{2p} v^q$ for σ_1, and monomials $t^{2q} v^{2p+1}$ or $t^{2q+1} v^{2p}$ for σ_2, with $p, q \in \mathbb{N}$. We will denote the number of such monomials by b_-, since they correspond to compact three-cycles $\mathcal{A}_k^-$ anti-invariant under σ^*. Respectively, the number of non-allowed monomials is denoted by b_+, and corresponds to the number of invariant compact three-cycles $\mathcal{A}_i^+$. For the local Calabi-Yau $Y(A_{2k}, m)$ and both involutions σ_1, σ_2 one has

$$m \text{ even:} \quad b_\pm = b/2 \, , \qquad\qquad m \text{ odd:} \quad b_\pm = \frac{1}{2}(b \mp k) \, , \tag{4.11}$$

where $b = mk(2k+1)$ is the total number of deformations in (4.9). The $\mathcal{N} = 1$ spectrum thus consists of b_- complex structure deformations S^k and b_+ U(1) vectors A^i_{H}. Finally, in accord with (3.10), the holomorphic three-form $\Omega = (du \wedge dv \wedge dt \wedge dw)/dP$, with P given in (4.8), transforms with a negative sign under the orientifold involutions.

4.3 The visible singularity

For the visible sector we will consider local del Pezzo surfaces introduced in section 3.3. In the following we will study del Pezzo singularities in compact Calabi-Yau manifolds, their resolutions and orientifold symmetries. In addition to general local considerations, we exemplify the techniques for an E_8 del Pezzo transition starting with the compact Calabi-Yau manifold $\mathbb{P}(18|9,6,1,1,1)$. Further examples are presented in section 5 and appendices A and B.

4.3.1 The geometry of Del Pezzo surfaces

Let us first discuss the geometry of del Pezzo surfaces $\mathcal{B}_n$ in more detail. This complements the analysis of section 3.3 and prepares us for the study of possible orientifold

$\mathfrak{g}$	del Pezzo	elliptic fibre
$\hat{D}_5$	$\mathbb{P}(2,2\|1,1,1,1,1)$	$\mathbb{P}(2,2\|1,1,1,1)$
$\hat{E}_6$	$\mathbb{P}(3\|1,1,1,1)$	$\mathbb{P}(3\|1,1,1)$
$\hat{E}_7$	$\mathbb{P}(4\|2,1,1,1)$	$\mathbb{P}(4\|2,1,1)$
$\hat{E}_8$	$\mathbb{P}(6\|3,2,1,1)$	$\mathbb{P}(6\|3,2,1)$

Table 4: The D-E del Pezzo surfaces and their generic elliptic fibers.

$\mathfrak{g}$	del Pezzo geometry
$\hat{D}_5$	$y^2 = x^2 + xe_1(w_1,w_2,z) + f_2(w_1,w_2)$ $y^2 = z^2 + zg_1(w_1,w_2,x) + h_2(w_1,w_2)$
$\hat{E}_6$	$y^3 = x^3 + xyg_1(w_1,w_2) + xf_2(w_1,w_2) + yg_2(w_1,w_2) + g_3(w_1,w_2)$
$\hat{E}_7$	$y^2 = x^4 + x^2g_2(w_1,w_2) + xg_3(w_1,w_2) + g_4(w_1,w_2)$
$\hat{E}_8$	$y^2 = x^3 + xg_4(w_1,w_2) + g_6(w_1,w_2)$

Table 5: The D-E del Pezzo surfaces.

involutions. Also here we will focus on the $\mathcal{B}_n$ with $n = 5,6,7,8$ with associated Lie groups $\mathfrak{g} = \hat{D}_5, \hat{E}_6, \hat{E}_7, \hat{E}_8$.[11] These del Pezzos are realized as hypersurfaces or complete intersections in weighted projective space. Moreover, they are elliptical models, with generic elliptic fiber realized as hypersurfaces or complete intersection. Let us denote by $\mathbb{P}(d_1,\ldots,d_r|w_0,\ldots,w_m)$ the complete intersection of r hypersurfaces of degree $d_1,\ldots,d_r$ in weighted projective space with weights $w_0,\ldots,w_m$. The del Pezzo surfaces and their generic elliptic fiber are listed in table 4.

As in sections 4.1 and 4.2 we can count the number of complex structure deformations associated to each generic del Pezzo singularity. table 4 allows us to specify a local equation with the singularity at the origin as displayed in table 5. One infers that the dimension of the complex deformation spaces for del Pezzo surfaces $\mathcal{B}_n$ with $n \geq 5$ is $\dim H^1(T\mathcal{B}_n) = 2n-8$. This can be checked, for example, for the $\hat{E}_7$ del Pezzo by counting $3+(4-2)+(5-1) = 6$ relevant monomials in g_2, g_3, g_4 respectively. For the $\hat{D}_5$ case one needs to consider vector polynomials.

4.3.2 The orientifold action on del Pezzo surfaces

On a compact Calabi-Yau space containing a del Pezzo surface, we are looking for a holomorphic involution σ obeying (3.10). In general, such involutions might not map the del Pezzo surfaces to its own homology class, but rather induce a more complicated action. This implies that $b^2_- \neq 0$, since there exist positive and negative linear combinations of

[11] Here the corresponding Lie algebras are denote with a hat in order to distinguish these singularities from the A-D-E singularities in section 4.2.

homologically non-trivial four-cycles. Examples of this type have been discussed e.g. in refs. [55]. Here we are interested in involutions σ preserving the non-trivial del Pezzo divisor in the compact space, but exclude the trivial case for which the del Pezzo is entirely in the fix-point set of σ. Via the proper transform σ acts uniquely on the coordinates of the del Pezzo surface. We are particularly interested in the action of the involutions on $H_2(S,\mathbb{Z})$ of the del Pezzo surface, which we call here generically S. Ideally we like to find invariant elements. Later on, for explicit compact examples, one needs to show that these are in the same homology class as the 2-cycles of the hidden singularity such that we can apply the mechanism of section 3.1 to obtain a non-Kähler space.

Let us work out the action on $H_2(S,\mathbb{Z})$ using the Lefschetz fixpoint theorem. The latter states that for an automorphism σ on any S the Lefschetz number Λ_σ equals the Euler number $\chi(S^\sigma)$ of the fixpoint set S^σ, i.e.

$$\chi(S^\sigma) = \Lambda_\sigma \equiv \sum_{k\geq 0} \mathrm{Tr}(\sigma^*(H_k(S,\mathbb{Z})) \ . \tag{4.12}$$

We can use this to determine the number of $\pm$ eigenvalues of σ^* on $H_2(S,\mathbb{Z})$. Clearly $H_0(S)$ and $H_4(S)$ are invariant and the non-trivial information comes from $H_2(S) = H_{(1,1)}(S)$. It follows that the number of positive and the negative eigenvalues on middle homology are

$$b_2^+ = \frac{n-1+\chi(S^\sigma)}{2}, \qquad b_2^- = \frac{3+n-\chi(S^\sigma)}{2} \tag{4.13}$$

Let us start with the classification of involutions. A theorem of [56] lists all pairs of minimal involutions (S,σ), where S is birational to $\mathbb{P}^2$. Lemma 1.1 of [56] states that minimality is equivalent to $\sigma(E) \neq E$ and $E \cap \sigma(E) \neq \emptyset$ for any exceptional curve E. In addition there are various non-minimal involutions on each del Pezzo surface.[12]

- $\hat{E}_8$ *del Pezzo:*

  Apart form an action on rational ruled surfaces one finds on the $\hat{E}_8$ del Pezzo the famous *Bertini involution* $y \mapsto -y$. The fixpoint set is a curve of genus $g = 4$ given by $\mathbb{P}(6|2,1,1)$ in the last three coordinates and the point $\mathbb{P}(6|3,2)$ so $\chi(S^\sigma) = 1+2-2g = -5$ and $b_2^- = 8$. K is invariant and, in fact, $-2K$ defines the hypersurface in $\mathbb{P}(3,2,1,1)$, such that that all of the E_8 lattice is mapped to $-E_8$. On the classes one has $e_i \mapsto -2K - e_i$ and $l \mapsto -l - 6K$. One can also see that the 240 rational curves table 2 are reflected on the middle configuration. The Bertini involution is the only minimal involution on the $\hat{E}_8$ del Pezzo.[13]

  As a non-minimal involution we can act by $w_1 \mapsto -w_1$. The fix point set is a curve of genus $g = 1$ given by $\mathbb{P}(6|3,2,1)$ and three points $\mathbb{P}(6|2,1)$. Hence $b_2^+ = 5$, $b_2^- = 4$.

- $\hat{E}_7$ *del Pezzo:*

[12] It turned out that automorphisms of del Pezzo surfaces have been recently investigated more extensively in the mathematical literature [57].

[13] The Bertini involution has striking analogy with the corresponding involution on the Enriques surface used in the orientifold model of ref. [58].

Very similar to the Bertini involution is the *Geiser involution* on the $\hat{E}_7$ del Pezzo which also acts like $y \mapsto -y$. The fixpoint set is a curve of genus $g = 3$ given by $\mathbb{P}(4|1,1,1)$ in the last three coordinates. This implies that $b_2^- = 7$, and since $-K$ is invariant, we have $E_7 \mapsto -E_7$, i.e. $e_i \mapsto -K - e_i$ and $l \mapsto -l - 3K$. Again the $7, 21, 21, 7$ rational curves in table 2, are reflected into themselves. The Geiser involution is the only minimal involution on the E_7 del Pezzo.

A non-minimal involution is obtained by the action $w_1 \mapsto -w_1$. It yields a genus $g = 1$ curve $\mathbb{P}(4|2,1,1)$ as fixpoint locus, and two points $\mathbb{P}(4|2,1)$. This yields $b_2^+ = 4$ and $b_2^- = 4$.

- $\hat{E}_6$ *del Pezzo:*

  Also on the $\hat{E}_6$ del Pezzo we can act with $y \mapsto -y$. However, there are no minimal involutions on this del Pezzo. The configuration $P = f_3(x, w_1, w_2) + y^2 f_1(x, w_1, w_2) = 0$, allowing this automorphism, is still generically smooth. By Bertinis theorem the non-vanishing of dP outside $(x : y : w_1 : w_2) = (0 : 0 : 0 : 0)$ has to be checked only on the base locus, which is given by all possible unions of the coordinate hyperplanes $x_i = 0$. The fixpoint locus is a genus $g = 1$ curve $\mathbb{P}(3|1,1,1)$ in the last three coordinates and the point $(1 : 0 : 0 : 0)$, which also lies on $P = 0$. Hence $b_2^+ = 3$ and $b_2^- = 4$.

  A second automorphism on the E_6 del Pezzo is given by $(y \mapsto -y, x \mapsto -x)$. The invariant equation $P = g_3(w_1, w_2) + x^2 f_1(w_1, w_2) + xyg_1(w_1, w_2) + y^2 h_1(w_1, w_2) = 0$ is likewise smooth as seen by checking transversality on the base locus. The fix point locus is given by $\mathbb{P}(3|1,1)$ in the first two coordinates, i.e. 3 points, and $w_1 = w_2 = 0$, which is a projective line $(y : x)$. Hence $b_2^+ = 5$ and $b_2^- = 2$.

- $\hat{D}_5$ *del Pezzo:*

  On the $\hat{D}_5$ del Pezzo we can act by $y \mapsto -y$ which has as a fixpoint set only the $g = 1$ curve $\mathbb{P}(2,2|1,1,1,1)$ in the last four coordinates, i.e. $b_2^+ = 2$ and $b_2^- = 4$. The configuration is smooth if not all 2×2 minors of $\frac{\partial P_i}{\partial x_j}$ vanish on the base locus unless $(x : y : z : w_1 : w_2) = (0 : 0 : 0 : 0 : 0)$. This is the case, if we chose for $P_1 = \sum_{i=1}^5 a_i x_i^2$ and $P_2 = \sum_{j=1}^5 b_j x_j^2$ with generic coefficients a_i, b_j

  A second involution acts as $(y \mapsto -y, x \mapsto -x)$. In this case we get the fixpoint locus $\mathbb{P}(2,2|1,1,1)$ in the last three coordinates, which are 4 points. We conclude then that $b_2^+ = 4$ and $b_2^- = 2$.

4.3.3 Creating a del Pezzo singularity

In order to realize a del Pezzo in a compact Calabi-Yau manifold we study a del Pezzo transition. Similar to the simple transitions of section 4.1 one starts by fixing a number of complex structure deformations to generate a del Pezzo singularity. There exist whole chains of such transitions, which are best understood in a toric framework as discussed in appendices A and B. Here we will study only one particular example [59], which is of relevance to the large volume compactifications of [60].

JHEP10(2008)077

Let us start with the an elliptic fibration over $\mathbb{P}^2$ which is represented as a degree 18 hypersurface in the weighted projective space $\mathbb{P}(9,6,1,1,1)$ as [61]

$$y^2 = x^3 + xf_{12}(\underline{w}) + f_{18}(\underline{w}) \,, \tag{4.14}$$

where $(\underline{x}) = (y, x, w_1, w_2, w_3)$ are the weighted coordinates. We can count the complex structure deformations modulo automorphism of the ambient space by enumerating the elements in $\mathcal{R} = \mathbb{C}^{\underline{w}}[\underline{x}]/\mathcal{J}$, where $\mathbb{C}^{\underline{w}}[\underline{x}]$ is the ring generated by all monomials in $\underline{x}$ and the left ideal $\mathcal{J}$ is generated by $\frac{\partial P}{\partial x_i}$, $i = 1, \ldots, 5$ with $P = y^2 - x^3 - xf_{12}(\underline{w}) - f_{18}(\underline{w})$. We see that y^2 is in the ideal, x^3 is modulo the ideal equivalent to xf_{12} with 91 monomials and from the 190 monomials in f_{18} the 9 of the form $w_i^{18}, w_i^{17}w_j$ are modulo the ideal equivalent to such with lower powers in w_i. This yields $h^{(2,1)} = 190 + 91 - 9 = 272$ complex structure deformations, which can be parameterized by the complex coefficients of the independent generators of $\mathcal{R}$. Note that different to the powers $w_i^{17}w_j$, but similar to the powers y^1, x^3 we write the powers of w_i^{18} with coefficient one to keep P transversal.

To force near the point $p = (0,0,0,0,1)$ inside $\mathbb{P}(18|9,6,1,1,1)$ a singularity of the form $\hat{E}_8$ in table 4 we need to set the coefficients of the homogeneous of degree 12 in $\underline{w}$ monomials $xw_1^m w_2^n w_3^k$ with $9 \le k \le 12$ to zero. This yields 10 conditions. Further we have to set the coefficients of the homogeneous degree 18 monomials $w_1^m w_2^n w_3^k$ in g_{18} for $13 \le k \le 18$ to zero. Naively these are 21 terms. However the 2 terms occurring for w_3^{17} are already set to zero as they are not independent modulo the ideal. The coefficient of the term w_i^{18} is constant in the standard parameterization of (4.14). We have to use a non standard parameterization for it to set it to zero. It can be easily shown that the elements of the ring that we are counting are still independent. So in total we can force the singularity by imposing $10 + 21 - 2 = 29$ conditions on the complex structure parameters.

In the next step one has to resolve the del Pezzo singularity by a finite-size del Pezzo surface. The difference in the resolution is that, unlike the surgery in codimension two encountered in sections 4.1 and 4.2, here we blow up a divisor and the triviality of the canonical bundle of the resolved space has to be checked explicitly.

4.3.4 Triviality of the canonical bundle in the resolution

Toric geometry is the standard and most general procedure to blow up the singularity at p and to check that the canonical class remains trivial. Let us first discuss a simpler class of examples, which includes the one of section 4.3.3, and consider a Calabi-Yau space M obtained as m-fold cover of a Fano threefold D [62]. Mori and Mukais classification list gives plenty of choices for D [63]. In an affine coordinate patch parameterized by y, x and $\underline{w}$ the defining equation of M is of the form

$$y^m = g(x, \underline{w}) \,. \tag{4.15}$$

We choose the complex structure parameters in $g(x, w)$ so that a del Pezzo singularity of the form $\hat{E}_6$, $\hat{E}_7$ or $\hat{E}_8$ occurs.

Now the blow up can be described as follows [62]. The branch locus B is defined as the zero locus of $g(x, \underline{w})$. Blowing up the singular point P at $x = \underline{w} = 0$ in B yields a complex two-dimensional exceptional divisor $E_D = \pi^{-1}(P)$ in the smooth Fano threefold $\hat{D}$. Here $\pi : \hat{D} \to D_s$ is the resolution map from the smooth to the singular variety[14]. The multiple covering of the exceptional divisor branched at the proper transform $\tilde{B}$ of B on E_D yields the blown-up surface E_M in the resolved Calabi-Yau threefold $\hat{M}$. As always the blow-up is local and holomorphic and does not change the complex structure. However, the exceptional divisor E_M in $\hat{M}$ is of codimension one and hence the canonical class K can be modified. If $\varphi : \hat{M} \to \hat{D}$ is the m-fold covering map, then the canonical class of the resolved manifold is

$$K_{\hat{M}} = \varphi^* \left(K_{\hat{D}} + \frac{m-1}{m} \tilde{B} \right) . \tag{4.16}$$

The calculation of $K_{\hat{D}}$ is easily done using the local geometry. The proper transform $\tilde{B}$ is given by $\tilde{B} = \pi^* B - \mathrm{ord}_P \; B \cdot E_D$, where ord is the order of vanishing of g in (4.15). Using (4.16) the condition $K_{\tilde{X}} = 0$ can be ensured in explicit examples.

In particular, in the transition of section 4.3.3 we have $m = 2$ as seen from (4.14). The blow up corresponds to a $\hat{E}_8$ del Pezzo and B is the projective bundle $B = \mathbb{P}(\mathcal{O}_{\mathbb{P}^2} \oplus \mathcal{O}_{\mathbb{P}^2}(-6))$. The upshot of the more general toric argument is that we blow up one divisor D_E in T_Δ by adding a point to the polyhedron Δ. The new Δ is still reflexive and the proper transform of the hypersurface constraint is the anti-canonical class in it. This implies that the canonical class of the resolved space is trivial which immediately generalizes to the complete chain of del Pezzo transitions.

4.4 Numerical changes of the Euler characteristic

The change of the Euler characteristic $\Delta\chi$ in the transition $M \to \hat{M}$ can be understand completely locally in terms of the singularity and in particular its Milnor number and the Euler number of the exceptional divisor E_M in $\hat{M}$. Let us discuss this here more systematically for the singularities encountered so far. For quasihomogenous surface singularities $f = 0$ the Milnor number was given in ref. [64] as $\mu = \dim \mathcal{O}/J_f$. If the leading terms are fermat, i.e. of the form $\sum_{i=1}^k x_i^{n_i}$, the Milnor number is given by

$$\mu = \prod_{i=1}^k (n_i - 1) . \tag{4.17}$$

The change of the Euler characteristic is determined as follows. Taking the singular point out of X the Euler characteristic changes by $\Delta\chi = \mu - 1$, while gluing in the exceptional divisor we get [62]

$$\Delta\chi = \mu - 1 + \chi(E_X) . \tag{4.18}$$

The change $\Delta\chi$ is easily determined for the examples we considered so far. For the node (4.3) one uses (4.17) to derive $\mu = 1$. So every blow-up increases the Euler characteristic by $\chi(\mathbb{P}^1) = 2$. One can also consider the blow-up of the A-D-E surface singularities in table 3. The Milnor number is $r = \mathrm{rank}(\mathfrak{g})$. In total, for the singular threefold (4.7) one derives $\mu = r(mh - 1)$ using (4.17). The resolving manifold is a $\mathbb{P}^1$-tree forming the Dynkin

[14] See also the discussion of (4.6) for the quintic.

diagram of $\mathfrak{g}$, its Euler number is $2 \cdot r - (r-1) = r+1$, where $r-1$ comes from subtracting the intersection points, hence $\Delta\chi = \mu - 1 + (r+1) = mhr$. For the A_r singularity we find by using table 3 that $\Delta\chi = m(r+1)r$. Indeed, $\Delta\chi/2$ is precisely the number of deformations of the A_r singularity in (4.9) as expected by geometric transition.

Most importantly, we can determine $\Delta\chi$ for the visible sector del Pezzo surfaces $\mathcal{B}_n$ listed in 4. For the $\hat{E}_6, \hat{E}_7, \hat{E}_8$ del Pezzos we calculate using 4 and (4.17) that $\mu = 16, 27, 50$. The Milnor number for the $\hat{D}_5$ del Pezzo is $\mu = 9$. Hence, given the Euler number $\chi(\mathcal{B}_n) = 3+n$, we get $\Delta\chi = 16, 24, 36, 60$ for $\mathcal{B}_n$, $n > 4$. In general, we can write

$$\Delta\chi = 2h(\mathfrak{g}), \tag{4.19}$$

where h is the dual Coxeter number of the algebra $\mathfrak{g}$. For the relevant cases $h(\mathfrak{g})$ is listed in table 3. We can now make use of $\Delta\chi$ to determine the rank of the map Π given in (3.35) which specifies the embedding of the del Pezzo surface $\mathcal{B}_n$ into a Calabi-Yau manifold. One first has to count the number of complex structure deformations $\Delta h^{(2,1)}$ which are fixed by specializing to a del Pezzo singularity such as the ones of table 5. The rank of Π is then simply given by $\mathrm{rank}(\Pi) = \frac{1}{2}\Delta\chi - \Delta h^{(2,1)}$.

5. Global engineering of U(1) mediation

In this section we explicitly construct orientifolds of compact internal manifolds which admit the desired properties to allow for U(1) mediation of supersymmetry breaking. We illustrate the general strategy by analyzing an explicit compact example permitting a visible E_6 del Pezzo surface and a hidden singularity with S^3's arising from deformations of conifold singularities. Many checks of the geometric requirements are best analyzed in a toric realization which we will provide in appendix A.2. It should be noted that similar examples can be constructed straightforwardly within the toric framework as we show in appendix A.3.

Let us first summarize the steps required in realizing the geometrical outset for scenarios of U(1) mediation:

(1) Find a *Calabi-Yau manifold with the desired singularities* for the hidden and visible sector. Ensure that the visible singularity can be resolved by, for example, a del Pezzo surface S as described in section 3.3 and 4.3. The hidden singularity can be of A-D-E type, and should contain conifolds which can be either resolved by two-spheres or deformed by three-spheres as discussed in sections 3.4 and 4.2.

(2) There should be *topological relations* between cycles in the hidden and visible sector. These topological relations can analyzed if the hidden and visible singularities are resolved by two-spheres and the del Pezzo surface respectively. More precisely, some of the two-cycles in the del Pezzo S should be non-trivial in the Calabi-Yau space and homologous to the hidden two-spheres. Upon geometric transition of some of the hidden two-spheres to three-spheres as in section 3.1, there are three- and four-chains connecting the hidden and visible sector.

(3) One needs to find an appropriate *geometrical orientifold symmetry* σ on the internal space. σ has to keep the del Pezzo class invariant. As discussed in section 4.3.2, there will be always σ-positive and negative two-cycles in S if σ is non-trivial on S. One of the positive two-cycles in S should be non-trivial in the Calabi-Yau space and in topological relation to the hidden two-spheres. Some of the hidden two-spheres are exchanged under σ such that hidden U(1) vectors arising upon geometric transition remain in the spectrum. Upon integrating out the massive moduli there will be light U(1) vector multiplets coupling to both the hidden and visible sector as discussed in section 2.2 and 3.3.

(4) *Local engineering of the background fluxes and gauge groups.* To make such a scenario fully realistic one needs to place appropriate branes on S to get an extension of the MSSM. In the hidden sector one performs a geometric transition of the resolving two-spheres and places appropriate fluxes on the resulting three-cycles to break supersymmetry. It has to be ensured that the scales are appropriate to yield an interesting set of soft terms for the MSSM sector.

In the following we will realize steps (1), (2) and (3) for one compact example. The geometric outset for two further examples are presented in appendices A.1 and A.3. Our first investigation will start from the compact Calabi-Yau obtained from a singular quintic as considered in ref. [65]. It is realized via the complete intersection

$$\begin{aligned} \big(P_1 s + P_2 t\big)\, u + \big(Q_1 s + Q_2 t\big)\, v &= 0\,, \\ \big(R_2 s + R_3 t\big)\, u + \big(S_2 s + S_3 t\big)\, v &= 0\,, \end{aligned} \tag{5.1}$$

in an ambient space defined as $\mathbb{P}^1 \times \mathbb{P}(\mathcal{O}_{\mathbb{P}^3}(-1) \oplus \mathcal{O}_{\mathbb{P}^3})$. Here P_i, Q_i, R_i and S_i are monomials of degree i in the complex projective coordinates $\underline{w} = (w_1, w_2, w_3, w_4)$ on the $\mathbb{P}^3$ base, while the coordinates on the $\mathbb{P}^1$ fibers are denoted by s, t. The $\mathbb{P}^1$ factor has coordinates u, v. The toric data of this Calabi-Yau manifold Y_{E_6} are summarized in appendix A.2. Y_{E_6} is a K3 fibration with

$$h^{(1,1)} = 3\,, \qquad h^{(2,1)} = 59\,, \qquad [c_2]_1 = 44\,, \qquad [c_2]_2 = 50\,, \qquad [c_2]_3 = 24\ . \tag{5.2}$$

where $[c_2]_i = \int_{Y_{E_6}} c_2 \wedge \omega_i$ with ω_i being a basis of $H^2(Y, \mathbb{Z})$. The non-vanishing triple intersections $\kappa_{ijk} = \int_{Y_{E_6}} \omega_i \wedge \omega_j \wedge \omega_k$ are computed to be

$$\kappa_{111} = 2, \qquad \kappa_{112} = \kappa_{122} = \kappa_{222} = 5, \qquad \kappa_{113} = 4, \qquad \kappa_{123} = \kappa_{223} = 6\ . \tag{5.3}$$

In the following we will check that Y_{E_6} admits in its moduli space two singularities of the desired type. This includes an $\hat{E}_6$ del Pezzo at $(s, t, u, v, \underline{w}) = (1, 0, 0, 0, \underline{0})$ as well as 32 conifold singularities. To see this one considers a point where at least one of the functions in front of u, v is non-zero and eliminates u, v in (5.1) to

$$\big(P_1 s + P_2 t\big)\big(S_2 s + S_3 t\big) - \big(R_2 s + R_3 t\big)\big(Q_1 s + Q_2 t\big) = 0 \tag{5.4}$$

The term in front of s^2 yields a non-generic cubic surface in coordinates $\underline{\omega}$ of $\mathbb{P}^3$ given by

$$P_1 S_2 - R_2 Q_1 = 0\ . \tag{5.5}$$

From table 4 we identify this as a non-generic $\hat{E}_6$ del Pezzo surface. To identify the 32 resolved conifolds we compare (5.4) and (5.1) to the simple singular conifold (4.4) and its resolution by a two-sphere (4.5). We see that the singular conifolds are obtained if all four factors in front of u, v in (5.1) vanish simultaneously. In fact, generically there are 32 conifold points where these vanishing conditions can be satisfied. At generic points on the moduli space Y_{E_6} is smooth and the conifolds are resolved by 32 two-spheres. The Calabi-Yau space Y_{E_6} can be obtained via geometric transition from a quintic surface by enforcing the $\hat{E}_6$ del Pezzo and 32 conifolds by fixing $\Delta h^{(2,1)} = 42$ of the 101 quintic complex structure deformations [65]. These transitions are discussed torically in appendix A.2.

In the next step we need to check that there is a two-cycle in the $\hat{E}_6$ del Pezzo $\mathcal{B}_6$ which is non-trivial in Y_{E_6} and in the same homology class as the 32 resolved conifolds. We will explicitly identify the two-cycles in $\mathcal{B}_6$ which are non-trivial in Y_{E_6}. Note that since $h^{(1,1)} = 3$ there are in total three non-trivial two-cycles in Y_{E_6}, which are denoted by Σ_B, Σ_K and Σ_α. The two-cycle $\Sigma_K \cong -K$ corresponds to the anti-canonical class of the $\hat{E}_6$ del Pezzo embedded into Y_{E_6}. In case this is the only class non-trivial in Y_{E_6} one will find 27 genus zero curves in its homology class. These are the 27 lines in table 2, which by (3.34) have genus zero and degree one with respect to $-K$. In fact, we can compute the genus zero BPS invariants $n_{n,m,l}$ by using the toric description of Y_{E_6} in appendix A.2. The three indices of $n_{m,n,l}$ indicate the degree of the curve with respect to Σ_K, Σ_B and Σ_α. If Σ_K is the only non-trivial class of $\mathcal{B}_6$ in Y_{E_6} one has $n_{1,0,0} = 27$. However, one instead computes

$$n_{1,0,0} = 10\,, \qquad n_{1,0,1} = 16\,, \qquad n_{1,0,2} = 1\,, \qquad\qquad n_{0,0,1} = 32\ . \tag{5.6}$$

Since Y_{E_6} can be realized torically as in appendix A.2 the BPS numbers can be obtained with the methods explained in refs. [66].[15] This shows that there is actually a second curve class $\Sigma_\alpha \cong \alpha$ in the del Pezzo surface which is non-trivial in Y_{E_6}. In accord with (5.6) one identifies

$$\alpha = 2l - e_1 - e_2 - e_5 - e_6\,, \qquad\qquad \alpha \cdot K = -2\ . \tag{5.7}$$

where l, e_i and K are introduced in section 3.3. Using (3.34) one checks that there are 10, 16 and 1 curves of table 2 with degree $0, 1$ and 2 with respect to α. We can write this as $\mathbf{27} = \mathbf{10}_0 + \mathbf{16}_1 + \mathbf{1}_2$, where the subscripts correspond to the degree with respect to α. This decomposition corresponds to the splitting of the $\mathbf{27}$ of E_6 into representations of $\mathrm{U}(1) \times D_5$. We can shift α by the other non-trivial class K such that the new linear combination is in the E_6 lattice. Defining $\alpha' = 3\alpha + 2K$ one has $K \cdot \alpha' = 0$. The new class α' induces the splitting

$$\mathbf{27} = \mathbf{10}_{-2} + \mathbf{16}_1 + \mathbf{1}_4\,, \tag{5.8}$$

where the subscripts indicate the degree with respect to α' and represent the U(1) charge in the usual normalization [67]. The last BPS invariant $n_{0,0,1}$ in (5.6) shows that there are actually 32 genus zero curves in the homology class of Σ_α. These are the 32 resolved conifolds discussed above. This establishes the desired topological relation of the del Pezzo to the candidate hidden singularities.

[15] The explicit calculations are implemented in the program Instanton, which uses the toric data as input parameters.

Let us now turn to the orientifold projection. As we have seen in section 4.3.2, there are two non-trivial orientifold projections on the $\hat{E}_6$ del Pezzo. One of them inverts two coordinates and thus, in order to ensure (3.10), would have to be accompanied by an inversion of an additional coordinate in the Calabi-Yau embedding. In order to avoid this complication, we will focus on the second involution which only inverts one coordinate, say $w_4 \to -w_4$ in (5.1). In general, a $\hat{E}_6$ del Pezzo surface admitting this involution can be brought to the form

$$w_4^2(\alpha\, w_1 + \beta\, w_2 + \gamma\, w_3) + w_1^3 + w_2^3 + w_3^3 + \delta\, \omega_1\omega_2\omega_3 = 0\ . \tag{5.9}$$

This del Pezzo can be brought to the non-generic form (5.5) by fixing two of the four complex structure deformations $\alpha, \beta, \gamma, \delta$. Slightly redefining coordinates one has

$$w_1(w_1^2 - 3w_1w_3 + \delta\, w_2w_3 + 3w_3^2) + w_2(w_2^2 - \delta\, w_3^2 + \beta\, w_4^2) = 0\ . \tag{5.10}$$

Next, we can determine a basis of $H_2^+(\mathcal{B}_6) \oplus H_2^-(\mathcal{B}_6)$. Recall from section 4.3.2 that $b_+^2 = 3$, $b_-^2 = 4$ for a del Pezzo surface with the above involutive symmetry. Since the canonical class of the del Pezzo is invariant under σ, one can embed the orientifold involution into the Weyl group of E_6. σ will be represented by four Weyl reflections $\sigma_a : \beta \mapsto \beta + (\beta\cdot r_a)\, r_a$ on four mutually orthogonal roots r_a [68]. Up to coordinate redefinitions one finds

$$\sigma^* = \prod_{a=1}^{4} \sigma_a\, , \qquad r_1 = \alpha_{\max}\, , \quad r_2 = \alpha_1\, , \quad r_3 = \alpha_3\, , \quad r_4 = \alpha_5\, , \tag{5.11}$$

where α_i are the simple roots defined in (3.32), and $\alpha_{\max} = 2l - \sum_i e_i$ is the maximal root of E_6. It is easy to check that the $\Sigma_a^- = r_a$ are a basis of $H_2^-(\mathcal{B}_6)$. Such a correspondence is a general fact for involutions in a Weyl group [68]. A basis of $H_2^+(\mathcal{B}_6)$ is given by $\Sigma_3^+ = -K$ as well as the two elements

$$\Sigma_1^+ = 2\alpha_2 + \alpha_1 + \alpha_3\, , \qquad \Sigma_2^+ = 2\alpha_4 + \alpha_3 + \alpha_5\ . \tag{5.12}$$

Finally, we can express the element α' given after (5.7) in this basis, $\alpha' = 2\Sigma_2^+ - 2\Sigma_1^+$. This shows that the second del Pezzo two cycle Σ_α, which is non-trivial in the full Calabi-Yau space, is also positive under the orientifold involution. Hence, for the Calabi-Yau orientifold space (Y_{E_6}, σ) we have $h_+^{(1,1)} = 3$, $h_-^{(1,1)} = 0$. The dimensions of $h_\pm^{(2,1)}$ can now be evaluated by using the Lefschetz fixpoint theorem as in (4.12).

In order that Y_{E_6} is a candidate to permit U(1) mediation, we have to show that there are actually hidden U(1) vectors in the spectrum. To do that, one shows that the 32 two-spheres in the homology class of Σ_α are not all mapped to themselves under σ. In fact, one checks that generically only two conifold points are invariant under $w_4 \to -w_4$, while the remaining 30 points are pairwise identified. This will remain to be the case if these singularities are resolved by two-spheres or deformed by three-spheres. Finally, one needs to proceed as in section 3.1 to construct the non-Kähler space $\mathcal{M}_6$. One performs a geometric transition replacing the 30 pairwise identified hidden two-spheres by three-spheres. Clearly, the resulting three-spheres will also be identified pairwise under the involution σ, such that several U(1) vectors from the R-R four-form remain in the spectrum. Since the two-spheres were in topological relation with the del Pezzo, the visible and hidden sector will be naturally connected via three- and four-chains in the non-Kähler space $\mathcal{M}_6$.

6. Conclusions

In this paper we investigated a promising mediation mechanism for supersymmetry breaking due to background fluxes in Type IIB string theory. Our motivation was the generic presences of U(1) vectors in semi-realistic string constructions of extended Supersymmetric Standard Models. We argued that under certain topological conditions these U(1) vector multiplets can couple to a hidden supersymmetry breaking flux geometry. Non-vanishing F-terms can render the gauginos massive which then induce phenomenologically interesting soft masses in the visible sector.

In order to find explicit realizations of such scenarios various requirements on the underlying compactification manifolds have to be met. Firstly, it should contain four-cycles on which intersecting space-time filling D-branes can provide an extension of the Standard Model. Promising candidates for such constructions are del Pezzo surfaces and we have provided a list of concrete compact Calabi-Yau manifolds admitting these as exceptional divisors. Secondly, the internal manifold should admit singularities which can support a supersymmetry breaking flux background. These are typically present in most of the known Calabi-Yau manifolds. However, the cycles in this hidden flux geometry have to be in topological relation with the visible sector geometry. In sections 4 and 5, we have summarized the general strategy to construct viable orientifold geometries for U(1) mediation.

Hypersurfaces and complete intersections provide a vast class of compact examples in which the required local geometries can be embedded and studied in detail. Combining the counting of the rational holomorphic curves in the compact Calabi-Yau manifold with the representation theory of the Weyl group acting on the curves in the del Pezzo surface, we were able to identify globally realized classes and their topological relations. Using the Lefschetz fixpoint theorem together with the embedding of the orientifold involution into the Weyl group, allows the determination of the parity of the curve classes. This provides the ground for explicit model building in type IIB orientifolds and F-theory. The described techniques are useful not only in modeling U(1) mediation. In particular, it should be possible to embed the geometric realizations of dynamical supersymmetry breaking [69] into the explicit Calabi-Yau orientifolds investigated in this work.

Let us comment on some of the open questions and directions for future work. In our investigation, we have not addressed the stabilization of moduli in detail. For type IIB orientifolds there are various known mechanism to render all geometric moduli massive. However, their stabilization might not entirely decouple from the low energy phenomenology investigated here [70]. The constructed compact orientifolds can, for example, support KKLT like vacua [39] or the large volume vacua studied in [60]. In particular, one can show that some of the examples realizing del Pezzo transitions as in section 4 and 5 have the desired properties for the large volume scenarios. It will be interesting to work out the explicit $\mathcal{N} = 1$ data for these examples and to study the supersymmetry breaking and moduli stabilization in more detail.

In order to get the correct scales of supersymmetry breaking it is often crucial to also include warping effects into the set-up. In particular, a hidden flux sector will generically break supersymmetry at a rather large scale. It is an interesting problem to study U(1) mediation of supersymmetry breaking in the presence of strongly warped throats. For this a better understanding of Kaluza-Klein reduction in warped backgrounds will be crucial.

Acknowledgments

We are grateful to Ralf Blumenhagen, Tae-Won Ha, Shamit Kachru, Denis Klevers, Eduard Looijenga, Dieter Lüst, Peter Mayr, Hans-Peter Nilles, Kyriakos Papadodimas and Alessandro Tomasiello for useful discussions. We in particularly thank Eduard Looijenga and Shamit Kachru for valuable correspondence. This work was partially supported by the European Union 6th framework program MRTN-CT-2004-503069 "Quest for unification", MRTN-CT-2004-005104 "ForcesUniverse", MRTN-CT-2006-035863 "UniverseNet" and SFB-Transregio 33 "The Dark Universe" by the DFG.

A. Toric realizations of the visible and the hidden singularities

A.1 K3 fibration with an $\hat{E}_6$ del Pezzo and 16 conifolds

In this appendix we will discuss the principles of toric transitions starting with the Calabi-Yau which is realized as the quintic hypersurface in $\mathbb{P}^4$. This manifold allows an $\hat{E}_6$ del Pezzo transition as well a transition through a singularity, which can serve as the hidden sector. Moreover, for the final Calabi-Yau there is a topological relation between a curve class on the del Pezzo surface, which serves as the visible sector, and a curve class in the hidden sector. This topological relation can be studied by computing the relevant BPS invariants and arguing as in section 5.

It turns out that such transitions are frequently realized for Calabi-Yau in toric ambient spaces. Let us recapitulate the essentials of the toric construction. We list in the following table in the first column the data of the reflexive polyhedra involved. More precisely the polyhedron Δ which yields $\mathbb{P}_\Delta = \mathbb{P}^4$, the ambient space of the quintic, is given by the convex hull of the first five points $\nu_1, \ldots, \nu_5$. If we add the point ν_6 we enforce transition I. We get as convex hull of the points $\nu_1, \ldots, \nu_6$ a polyhedron $\Delta^{(2)}$, which defines as $\mathbb{P}_{\Delta^{(2)}}$ the ambient space of a Calabi-Yau with an $\hat{E}_6$ del Pezzo. Similarly, we can consider the convex hull of the points $\nu_1, \ldots, \nu_5, \nu_7$ to enforce transition II. Finally, if we consider all 7 points $\nu_1, \ldots, \nu_7$ we are dealing with transition III in the table. In the following we will discuss these transitions in more detail.

As usual we define $\mathbb{P}_\Delta$ by the Cox coordinate ring as $\mathbb{P}_\Delta = \{C[x_1, \ldots, x_n] - SR\}/(\mathbb{C}^*)^r$. Here the $\mathbb{C}^*$ actions are specified by the vectors $l_k^{(p)}$, $k = 1, \ldots, r$ as $x_i \mapsto (\mu_k)^{l_{k,i}^{(p)}} x_i$, with $\mu_k \in \mathbb{C}^*$. The SR is the Stanley Reisner ideal, which is also specified by the $l_k^{(p)}$, see [71] for details. The index (p) on $l_i^{(p)}$ labels the Calabi-Yau phase under consideration after the respective transition. Note that the $l_k^{(p)}$ encode relations among the points in the polyhedron namely $\sum_{i=1}^n l_{k,i}^{(p)} \nu_i = 0$, where n is the number of relevant points. Using suitable triangulations of the polyhedron we have specifically chosen them so that in the Calabi-Yau phase indexed by (p) to every $l_k^{(p)}$, $k = 1, \ldots, h^{(1,1)}$ there is an uniquely associate Kähler modulus v^k of a curve in the Calabi-Yau, so that the Kähler cone is given by $v^k > 0$.

	quintic		transition I			transition II			transition III			
		$l^{(1)}$		$l_1^{(2)}$	$l_2^{(2)}$		$l_1^{(3)}$	$l_2^{(3)}$		$l_1^{(4)}$	$l_2^{(4)}$	$l_3^{(4)}$
points in polyhedra	x_0	-5		-3	-2	x_0	-4	-1	x_0	-2	-2	-1
ν_1 =(-1, 0, 0, 0)	x_1	1	x_1	1	0	u_1	0	1	u_1	0	0	1
ν_2 =(0, -1, 0, 0)	x_2	1	x_2	1	0	v_1	1	0	v_1	1	0	0
ν_3 =(0, 0, -1, 0)	x_3	1	x_3	1	0	v_2	1	0	v_2	1	0	0
ν_4 =(0, 0, 0, -1)	x_4	1	s	0	1	v_3	1	0	s	0	1	0
ν_5 =(1, 1, 1, 1)	x_5	1	x_4	1	0	u_2	0	1	u_2	0	0	1
ν_6 =(0, 0, 0, 1)			t	-1	1				t	-1	1	0
ν_7 =(0, 1, 1, 1)						r	1	-1	r	1	0	-1

Table 6: Transitions from the quintic to a K3 fibration with $\hat{E}_6$ del Pezzo and 16 conifolds.

The hypersurfaces or complete intersections represent the anti-canonical class of $\mathbb{P}_\Delta$ and are easily defined by the $l_k^{(p)}$, $k = 1, \ldots, r$. In the case of hypersurfaces the polynomial $P(x)$, whose zero section defines the Calabi-Yau in the coordinates $x_1, \ldots, x_m$ is simply such that $x_0 P(x)$ is totally invariant under the $(\mathbb{C}^*)^r$ actions.

Each point point of Δ corresponds to a toric divisor given as the zero locus of the corresponding coordinate, e.g. ν_1 corresponds to $x_1 = 0$ also called D_1. Among the divisors there are relations $\sum_{i=1}^n \nu_{i,k} D_i = 0$, where $\nu_{i,k}$ is the k component of ν_i. The Chern class of the ambient space is $c(T\mathbb{P}_\Delta) = \prod_{i=1}^n (1+D_i)$, the canonical class is $K = \sum_{i=1}^n D_i$. The total Chern class of the Calabi-Yau M, which is specified by $-K$, is $c(TM) = c(T\mathbb{P}_\Delta)/(1+K)$. The k-th Chern class c_k are then the terms homogeneous of order k in the D_i in the formal expansion of $c(TM)$. So by construction $c_1(TM) = 0$.

In the formalism of reflexive pairs (Δ, Δ^*) the transitions are very easily understood. Points in Δ count the Kähler parameter, while points in Δ^* counts complex parameters. If we add the point ν_6 to enlarge $\Delta = \Delta^{(1)}$ to $\Delta^{(2)}$, we create a del Pezzo singularity in the first 4 coordinates, because the enlargement of Δ enforces the vanishing of 11 coefficients of the monomials, which correspond to complex structure deformations in the Newton polynom of Δ^*. So $\Delta^{(2)} \supset \Delta$, which counts the Kähler parameter, becomes larger while its dual $\Delta^{(2)*} \subset \Delta^*$, which counts the complex parameters, becomes smaller.

In the table we have chosen coordinates which are most intuitive to understand the precise form of the polynomial constraint after the different transitions. For the quintic $P(\underline{x})$ is simply given by a homogeneous polynomial of degree five in the first five variables. Then after the transition I the constraint equation looks like

$$P = x_0 \sum_{k=0}^{2} p_{3+k}(\underline{x}) s^{2-k} t^k \ . \tag{A.1}$$

Here the $p_k(\underline{x})$ are homogeneous polynomials of degree k in $x_1, \ldots, x_4$. It is obvious that at $t = 0$ and $s = 1$ we get a E_6 del Pezzo singularity. General s, t with the above $\mathbb{C}^*$ actions correspond to the blow up of the del Pezzo singularity. Adding $l^{(1)} = l_1^{(2)} + l_2^{(2)}$, dropping

JHEP10(2008)077

the t variable which scales trivially under $l^{(1)}$, and identifying s with x_5 allows to deform the del Pezzo singularity to a generic quintic.

Let us analyze the Calabi-Yau obtained after transition I in more detail. As seen from $l^{(2)}$ the toric variety is $\mathbb{P}(\mathcal{O}_{\mathbb{P}^3} \otimes \mathcal{O}(-K)_{\mathbb{P}^3})$. We have the following topological data for the manifold: $\chi = -176$, $h^{(1,1)} = 2$ and

$$[c_2]_1 = 44\,, \quad [c_2]_2 = 50\,, \quad \kappa_{111} = 2\,, \quad \kappa_{112} = \kappa_{122} = \kappa_{222} = 5\,, \tag{A.2}$$

where $[c_2]_i = \int_M c_2 \wedge \omega_i$ and $\kappa_{ijk} = \int_M \omega_i \wedge \omega_j \wedge \omega_k$ as in (5.2) and (5.3). It is also instructive to list the genus zero BPS invariants for the rational curves $n_{i,j}$, because we see here the 27 lines of the $\hat{E}_6$ del Pezzo in the class $(1,0)$, which corresponds to the class $-K$.

$\underline{n_{i,j} \text{ for trans. I :}}$

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	60	0	0	0	0	0
1	**27**	2515	12210	12210	2515	27	0

Let us next turn to the transition II listed in the table above. After this transition the polynomial is of the form

$$P = x_0 \sum_{k=0}^{4} r^k p_{4-k}(\underline{v}) q_{k+1}(\underline{u}) \;. \tag{A.3}$$

We have now a singularity at $r = 0$, which is a $\mathbb{P}^1$ bundle over a curve of genus 3. It has been analyzed in [61, 72], where it has been argued that it can be deformed to isolated 16 $\mathbb{P}^1$, i.e. 16 conifolds. This manifold is a $K3$ fibration with the topological data $\chi = -168$, $h^{(1,1)} = 2$ and

$$[c_2]_1 = 50\,, \quad [c_2]_2 = 24\,, \quad \kappa_{111} = 5\,, \quad \kappa_{112} = 4\;. \tag{A.4}$$

For this Calabi-Yau manifold the BPS instantons are as follows

$\underline{n_{i,j} \text{ for trans. II :}}$

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	**16**	0	0	0	0	0
1	640	2144	120	-32	3	0	0

In particular we see that $n_{0,1} = 16$ which corresponds to the configuration which can be deformed to 16 conifolds.

Finally, we turn to the transition III for which both points ν_6, ν_7 are added. This leads to a polynomial of the form

$$P = x_0 \sum_{k=0}^{2} t^k s^{2-k} \sum_{l=0}^{k+2} r^l p_{2+k-l}(\underline{u}) q_{l+1}(\underline{v}) \;. \tag{A.5}$$

We see that at $t = 0$, $s = 1$ there is a non-generic del Pezzo singularity, while at $r = 0$ there is a degenerate version of the $\mathbb{P}^1$ bundle over a curve of genus 3, which can be deformed to 12 isolated $\mathbb{P}^1$'s, i.e. 12 conifolds. Again this manifold is a $K3$ fibrations with

$$\chi = -152\,, \quad h^{(1,1)} = 3\,, \quad [c_2]_1 = 24, \quad [c_2]_2 = 50\,, \quad [c_2]_3 = 44\,, \tag{A.6}$$

		quintic		trans. I		trans. II		trans. III		
			$l^{(1)}$	$l_1^{(2)}$	$l_2^{(2)}$	$l_1^{(3)}$	$l_2^{(3)}$	$l_1^{(4)}$	$l_2^{(4)}$	$l_3^{(4)}$
		x'_0				-3	-1	-2	-1	-1
		x_0	-5	-3	-2	-2	-1	-1	-1	-1
ν_1	=(-1, 0, 0, 0 0)	w_1	1	1	0	1	0	1	0	0
ν_2	=(0, -1, 0, 0 0)	w_2	1	1	0	1	0	1	0	0
ν_3	=(0, 0, -1, 0 0)	w_3	1	1	0	1	0	1	0	0
ν_4	=(0, 0, 0, -1 0)	s	1	0	1	1	0	0	1	0
ν_5	=(1, 1, 1, 1 0)	w_4	1	1	0	1	0	1	0	0
ν_6	=(0, 0, 0, 1 0)	t		-1	1			-1	1	0
ν_7	=(0, 0, 0, 0 1)	u				0	1	0	0	1
ν_8	=(0, 0, 0, 0 -1)	v				0	1	0	0	1

Table 7: Transitions from the quintic to a K3 fibration with $\hat{E}_6$ del Pezzo and 32 conifolds.

and the non-vanishing triple intersections

$$\kappa_{111} = \kappa_{113} = 2\,, \quad \kappa_{112} = \kappa_{122} = \kappa_{222} = 5\,, \quad \kappa_{123} = \kappa_{223} = 4\,. \tag{A.7}$$

The genus zero BPS instantons $n_{0,i,j}$, $n_{1,i,j}$ and $n_{2,i,j}$ are given by

$n_{0,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	**12**	0	0	0	0	0
0	50	12	-2	0	0	0	0

$n_{1,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	**10**	**16**	**1**	0	0	0	0
1	540	1920	76	-24	3	0	0
2	1396	10064	1035	-440	198	-48	0

A.2 K3 fibration with $\hat{E}_6$ del Pezzo and 32 conifolds

In this appendix we present the toric analysis of the Calabi-Yau space used in section 5. The geometric construction of this example has been discussed by Malyshev in ref. [65]. Our investigation will follow a similar logic as in appendix A.1 and we will likewise start with the quintic in $\mathbb{P}^4$. Again in the table polyhedron of $\mathbb{P}^4$ is the four simplex given the convex hull by the points $\nu_1, \ldots, \nu_5$ in $\mathbb{R}^4$, i.e. with the last entry dropped. The transition I to the Calabi-Yau with the $\hat{E}_6$ del Pezzo singularity is exactly as in appendix A.1. Since we like to study latter a transition to a complete intersection in a five dimensional space given by a five dimensional polyhedron we added a trivial fifths coordinate to the first six points.

Let us turn to the transition II. The $l^{(3)}$ are the scalings for a conifold transitions after generating 36 nodes. The two polynomials P and P' are defined by the $l^{(3)}$ requiring that $x_0 P$ and $x'_0 P$ are invariant under the $(\mathbb{C}^*)^2$ actions. The same is true for the transition

III. We have the following topological data for the manifold after the transition II. It is a $K3$ fibration with $\chi = -128$, $h^{(1,1)} = 2$ and

$$[c_2]_1 = 50\,, \quad [c_2]_2 = 24\,, \quad \kappa_{111} = 5\,, \quad \kappa_{112} = 6\,, \quad \kappa_{122} = \kappa_{222} = 0\,, \tag{A.8}$$

with a notation as in (5.2) and (5.3). From the genus zero BPS invariants for the rational we see the homologous 36 $\mathbb{P}^1$ blow-ups of the nodes in the class $(0, 1)$ as expected.

$n_{i,j}$ for trans. II :

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	**36**	0	0	0	0	0
1	366	1584	909	16	0	0	0
2	2670	73728	255960	231336	45216	360	-20

Finally, we can analyze the transition III. The resulting Calabi-Yau manifold and with a candidate orientifold projection has been studied in section 5. In the toric set-up we now have the scalings $l^{(4)}$ which correspond to performing both transitions simultaneously. We get a $K3$ fibration with topological data as in (5.2) and non-vanishing triple intersections (5.3). The BPS invariants $n_{0,i,j}$, $n_{1,i,j}$ and $n_{2,i,j}$ are given by

$n_{0,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	**32**	0	0	0	0	0
0	28	32	0	0	0	0	0

$n_{1,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	**10**	**16**	**1**	0	0	0	0
1	310	1408	781	16	0	0	0
2	310	4064	6418	1408	10	0	0

A.3 Example with $\hat{E}_7$ and $\hat{E}_6$ del Pezzo

In this appendix we present a compact Calabi-Yau example with an $\hat{E}_7$ and $\hat{E}_6$ del Pezzo which are in topological relation by intersecting non-trivially. We also start with the quintic and add the point ν_6 as in the following table.

At $t = 0$ we get an $\hat{E}_7$ del Pezzo. The topological data after the transition I are $\chi = -164$, $h^{(1,1)} = 2$ and

$$[c_2]_1 = 42\,, \quad [c_2]_2 = 50\,, \quad \kappa_{111} = 3, \quad \kappa_{112} = \kappa_{122} = \kappa_{222} = 5\ . \tag{A.9}$$

In the first table summarizing the BPS instantons we see $n_{1,0} = 56$, which indicates the class with the 56 lines of the $\hat{E}_7$ del Pezzo.

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	20	0	0	0	0	0
1	**56**	2635	5040	190	-40	3	0

After the transition II to the Calabi-Yau defined by $l_k^{(3)}$ we have an elliptic fibration with $\chi = -168$, $h^{(1,1)} = 2$ and

$$[c_2]_1 = 50\,, \quad [c_2]_2 = 36, \quad \kappa_{111} = \kappa_{112} = 5, \qquad \kappa_{122} = 3\ . \tag{A.10}$$

We find the BPS number $n_{0,1} = 18$ in the following table

		quintic		transition I			transition II			transition III			
			$l^{(1)}$		$l_1^{(2)}$	$l_2^{(2)}$		$l_1^{(3)}$	$l_2^{(3)}$		$l_1^{(4)}$	$l_2^{(4)}$	$l_3^{(4)}$
			-5	x_0	-4	-1	x_0	-3	-2	x_0	-2	0	-1
ν_1	=(-1, 0, 0, 0)	x_1	1	u	2	-1	u_1	0	1	u_1	1	0	-1
ν_2	=(0, -1, 0, 0)	x_2	1	w_1	1	0	u_2	0	1	v_1	0	1	0
ν_3	=(0, 0, -1, 0)	x_3	1	w_2	1	0	v_1	1	0	v_2	1	-1	0
ν_4	=(0, 0, 0, -1)	x_4	1	w_3	1	0	u_3	0	1	s	0	1	0
ν_5	=(1, 1, 1, 1)	x_5	1	s	0	1	v_2	1	0	u_2	0	0	1
ν_6	=(-2, -1, -1, -1)			t	-1	1				t	-1	1	1
ν_7	=(-1, -1, 0, -1)						r	1	-1	r	1	-2	0

Table 8: Transitions from the quintic to a CY threefold with $\hat{E}_6$ and $\hat{E}_7$ del Pezzos.

$n_{i,j}$ for trans. II :

i	j=0	j=1	j=2	j=3	j=4	j=5	j=6
0	0	**18**	-2	0	0	0	0
1	186	2439	442	-512	768	-1024	1280

In the following we will generate an $\hat{E}_7$ and $\hat{E}_6$ del Pezzo by considering both transitions simultaneously. After this transition III we find likewise an elliptically fibered Calabi-Yau space

$$\chi = -148\,, \quad h^{(1,1)} = 3\,, \quad [c_2]_1 = 78, \quad [c_2]_2 = 36, \quad [c_2]_3 = 50\,, \tag{A.11}$$

and the non-vanishing triple intersections

$$\begin{aligned} &\kappa_{111} = 21\,, \quad \kappa_{112} = 9\,, \quad \kappa_{122} = \kappa_{223} = 3\,, \quad \kappa_{113} = 18\,, \\ &\kappa_{123} = 8\,, \quad \kappa_{133} = 10\,, \quad \kappa_{233} = \kappa_{333} = 5\;. \end{aligned} \tag{A.12}$$

The BPS invariants $n_{0,i,j}$, $n_{1,i,j}$, $n_{2,i,j}$ and $n_{3,i,j}$ are given by

$n_{0,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4
0	0	16	0	0	0
1	0	0	0	0	0

$n_{1,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4
0	**12**	154	16	-2	0
1	**10**	**16**	**1**	0	0

$n_{2,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4
0	-2	16	154	12	0
1	**32**	2279	4392	130	-32
2	-2	-32	-20	0	0

$n_{3,i,j}$ for trans. III :

i	j=0	j=1	j=2	j=3	j=4
0	0	0	16	148	16
1	-64	528	48158	86440	4291
2	**12**	346	2600	1794	128

Here the 56 of E_7 which appeared after transition I in the class $(1,0)$ is now splitted by the class $(1,1,0)$, i.e. $n_{1,0,0} = 12$, $n_{2,1,0} = 32$ and $n_{3,2,0} = 12$. This corresponds to the splitting $\mathrm{SO}(12) \times \mathrm{SU}(2) \subset E_7$ under which the representation $\mathbf{56}$ decomposes into $\mathbf{56} = (\mathbf{12}, \mathbf{2}) + (\mathbf{32}, \mathbf{1})$ [67]. In the class $(1,1,0)$ are $n_{1,1,0} = 10$ rational curves. As can be seen from the above table these seem to be part of the $\mathbf{27} = \mathbf{10} + \mathbf{16} + \mathbf{1}$ curves of an $\hat{E}_6$ del Pezzo singularity which provides in this example the hidden singularity.

B. Del Pezzo transitions $M^{\mathfrak{g}}_k \to M^{\mathfrak{g}}_{k+1}$

In section 4.3 we briefly discussed a del Pezzo transition starting with the hypersurface $\mathbb{P}(18|9,6,1,1,1)$. Such del Pezzo transitions are in fact ubiquitous in toric Calabi-Yau transitions. In particular, there are whole chains of transitions in which up to five $\hat{E}_6, \hat{E}_7$ or $\hat{E}_8$ del Pezzo surfaces can be blown up respectively. The corresponding discrete family of 18 reflexive polyhedra $\Delta^{\mathfrak{g}}_n$ in $4d$ is the the convex hull of the points

$$\begin{array}{cccc} (\ -1, & 0, & 0, & 0) \\ (\ \ 0, & -1, & 0, & 0) \\ (m^{\mathfrak{g}}_1, & m^{\mathfrak{g}}_2, & 1, & 0) \\ (m^{\mathfrak{g}}_1, & m^{\mathfrak{g}}_2, & 0, & 1) \\ (m^{\mathfrak{g}}_1, & m^{\mathfrak{g}}_2, & \nu^{(0)}_1, & \nu^{(0)}_2) \\ (m^{\mathfrak{g}}_1, & m^{\mathfrak{g}}_2, & \ddots, & \vdots) \\ (m^{\mathfrak{g}}_1, & m^{\mathfrak{g}}_2, & \nu^{(n)}_1, & \nu^{(n)}_2) \end{array} \quad \text{with} \quad \begin{array}{l} \underline{\nu}^{(0)} = (-1,-1) \\ \underline{\nu}^{(1)} = (0,-1) \\ \underline{\nu}^{(2)} = (-1,0) \\ \underline{\nu}^{(3)} = (1,1) \\ \underline{\nu}^{(4)} = (1,-1) \\ \underline{\nu}^{(5)} = (-1,1) \end{array} \quad \text{and} \quad \begin{array}{l} \underline{m}^{E_8} = (3,2) \\ \underline{m}^{E_7} = (2,1) \\ \underline{m}^{E_6} = (1,1)\ . \end{array} \tag{B.1}$$

For $\mathrm{rank}(\mathfrak{g}) > 5$ these $d = 4$ polyhedra define hypersurfaces specified by the anti-canonical bundle in the in toric variety and thus appear in the list [73]. For $\mathrm{rank}(\mathfrak{g}) < 6$ one finds complete intersections associated to the nef partitions of toric varieties associated to reflexive polyhedra with $d > 4$.

The to (B.1) corresponding compact Calabi-Yau manifolds are generically smooth elliptic fibrations over a del Pezzo base. In this fibration the worst degeneration of the fiber is of Kodaira type I_1 [74]. Let us denote the type of the elliptic fibration by the Lie algebra $\mathfrak{g}$ and the corresponding elliptic fibered Calabi-Yau over $\mathcal{B}_n$ as $M^{\mathfrak{g}}_n$. As an collorary to the analysis of the elliptically fibered Calabi-Yau fourfolds [75], one finds that the Euler number of the elliptic fibration $M^{\mathfrak{g}}_n$ is given by

$$\chi(M^{\mathfrak{g}}_n) = -2h(\mathfrak{g}) \int_{\mathcal{B}_n} c_1^2 = 2h(\mathfrak{g})(n-9), \tag{B.2}$$

here $h(\mathfrak{g})$ is the coxeter number of the associated Lie algebra. $h(\mathfrak{g})$ has been listed in table 3. For the smooth fibration spaces just described, $n+1$ Kähler classes of $M_n^{\mathfrak{g}}$ come from the del Pezzo base $\mathcal{B}_n$ and $8-\text{rank}(\mathfrak{g})$ come from the $8-\text{rank}(\mathfrak{g})$ sections of the elliptic fiber. Using $\chi = 2(h^{(1,1)} - h^{(2,1)})$ one infers

$$\begin{aligned} h^{(1,1)}(M_n^{\mathfrak{g}}) &= n + 10 - \text{rank}(\mathfrak{g})\,, \\ h^{(2,1)}(M_n^{\mathfrak{g}}) &= h(\mathfrak{g})(9-n) + n + 10 - \text{rank}(\mathfrak{g})\ . \end{aligned} \tag{B.3}$$

Many of the $M_n^{\mathfrak{g}}$ have already appeared in the physics literature as, for example, in refs. [59, 76, 77]. In section 4.3 we discussed the transition $M_0^{E_8} \ \rightarrow \ M_1^{E_8}$.

References

[1] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71 [hep-th/0502005];
R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional String Compactifications with D-branes, Orientifolds and Fluxes*, *Phys. Rept.* **445** (2007) 1 [hep-th/0610327];
F. Marchesano, *Progress in D-brane model building*, *Fortschr. Phys.* **55** (2007) 491 [hep-th/0702094], and references therein.

[2] M.R. Douglas and G.W. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[3] G. Aldazabal, L.E. Ibáñez, F. Quevedo and A.M. Uranga, *D-branes at singularities: a bottom-up approach to the string embedding of the standard model*, *JHEP* **08** (2000) 002 [hep-th/0005067];
D. Berenstein, V. Jejjala and R.G. Leigh, *The standard model on a D-brane*, *Phys. Rev. Lett.* **88** (2002) 071602 [hep-ph/0105042];
H. Verlinde and M. Wijnholt, *Building the standard model on a D3-brane*, *JHEP* **01** (2007) 106 [hep-th/0508089];
M. Wijnholt, *Geometry of particle physics*, hep-th/0703047.

[4] R. Donagi and M. Wijnholt, *Model building with F-theory*, arXiv:0802.2969;
C. Beasley, J.J. Heckman and C. Vafa, *GUTs and exceptional branes in F-theory — I*, arXiv:0802.3391;
H. Hayashi, R. Tatar, Y. Toda, T. Watari and M. Yamazaki, *New aspects of heterotic-F theory duality*, arXiv:0805.1057.

[5] For a review see, for example G.F. Giudice and R. Rattazzi, *Theories with gauge-mediated supersymmetry breaking*, *Phys. Rept.* **322** (1999) 419 [hep-ph/9801271].

[6] For reviews see, for example H.P. Nilles, *Supersymmetry, supergravity and particle physics*, *Phys. Rept.* **110** (1984) 1;
D.J.H. Chung et al., *The soft supersymmetry-breaking Lagrangian: theory and applications*, *Phys. Rept.* **407** (2005) 1 [hep-ph/0312378].

[7] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B 557** (1999) 79 [hep-th/9810155];
G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027 [hep-ph/9810442].

JHEP10(2008)077

[8] For reviews on Z' physics see, for example A. Leike, *The phenomenology of extra neutral gauge bosons*, *Phys. Rept.* **317** (1999) 143 [hep-ph/9805494];
P. Langacker, *The physics of heavy Z' gauge bosons*, arXiv:0801.1345.

[9] B.A. Dobrescu, *B-L mediated supersymmetry breaking*, *Phys. Lett.* **B 403** (1997) 285 [hep-ph/9703390];
D.E. Kaplan, F. Lepeintre, A. Masiero, A.E. Nelson and A. Riotto, *Fermion masses and gauge mediated supersymmetry breaking from a single* U(1), *Phys. Rev.* **D 60** (1999) 055003 [hep-ph/9806430];
H.-C. Cheng, B.A. Dobrescu and K.T. Matchev, *A chiral supersymmetric standard model*, *Phys. Lett.* **B 439** (1998) 301 [hep-ph/9807246]; *Generic and chiral extensions of the supersymmetric standard model*, *Nucl. Phys.* **B 543** (1999) 47 [hep-ph/9811316];
L.L. Everett, P. Langacker, M. Plümacher and J. Wang, *Alternative supersymmetric spectra*, *Phys. Lett.* **B 477** (2000) 233 [hep-ph/0001073];
Y. Nakayama, *Stable SUSY Breaking Model with* O(10) *eV gravitino from combined D-term gauge mediation and* $\mathrm{U}(1)'$ *mediation*, *JHEP* **02** (2008) 013 [arXiv:0712.0619].

[10] P. Langacker, G. Paz, L.-T. Wang and I. Yavin, *Z'-mediated supersymmetry breaking*, *Phys. Rev. Lett.* **100** (2008) 041802 [arXiv:0710.1632];
P. Langacker, G. Paz, L.-T. Wang and I. Yavin, *Aspects of Z'-mediated Supersymmetry Breaking*, *Phys. Rev.* **D 77** (2008) 085033 [arXiv:0801.3693].

[11] R. Dermisek, H. Verlinde and L.-T. Wang, *Hypercharged anomaly mediation*, *Phys. Rev. Lett.* **100** (2008) 131804 [arXiv:0711.3211].

[12] H. Verlinde, L.-T. Wang, M. Wijnholt and I. Yavin, *A higher form (of) mediation*, *JHEP* **02** (2008) 082 [arXiv:0711.3214].

[13] E. Witten, *New issues in manifolds of* SU(3) *holonomy*, *Nucl. Phys.* **B 268** (1986) 79.

[14] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [*Erratum ibid.* **608** (2001) 477] [hep-th/9906070];
T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[15] For reviews see, for example M.R. Douglas and S. Kachru, *Flux compactification*, *Rev. Mod. Phys.* **79** (2007) 733 [hep-th/0610102];
M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[16] M. Graña, *MSSM parameters from supergravity backgrounds*, *Phys. Rev.* **D 67** (2003) 066006 [hep-th/0209200];
P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241];
M. Graña, T.W. Grimm, H. Jockers and J. Louis, *Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes*, *Nucl. Phys.* **B 690** (2004) 21 [hep-th/0312232];
D. Lüst, S. Reffert and S. Stieberger, *Flux-induced soft supersymmetry breaking in chiral type IIB orientifolds with D3/D7-branes*, *Nucl. Phys.* **B 706** (2005) 3 [hep-th/0406092].

[17] K. Choi, A. Falkowski, H.P. Nilles and M. Olechowski, *Soft supersymmetry breaking in KKLT flux compactification*, *Nucl. Phys.* **B 718** (2005) 113 [hep-th/0503216];
M. Endo, M. Yamaguchi and K. Yoshioka, *A bottom-up approach to moduli dynamics in heavy gravitino scenario: superpotential, soft terms and sparticle mass spectrum*, *Phys. Rev.* **D 72** (2005) 015004 [hep-ph/0504036];

K. Choi and H.P. Nilles, *The gaugino code*, *JHEP* **04** (2007) 006 [hep-ph/0702146].

[18] S. Kachru, L. McAllister and R. Sundrum, *Sequestering in string theory*, *JHEP* **10** (2007) 013 [hep-th/0703105].

[19] M. Aganagic, C. Beem, J. Seo and C. Vafa, *Geometrically induced metastability and holography*, *Nucl. Phys.* **B 789** (2008) 382 [hep-th/0610249];
J.J. Heckman, J. Seo and C. Vafa, *Phase structure of a brane/anti-brane system at large-N*, *JHEP* **07** (2007) 073 [hep-th/0702077];
J.J. Heckman and C. Vafa, *Geometrically induced phase transitions at large-N*, *JHEP* **04** (2008) 052 [arXiv:0707.4011].

[20] K. Choi and K.S. Jeong, *Supersymmetry breaking and moduli stabilization with anomalous* U(1) *gauge symmetry*, *JHEP* **08** (2006) 007 [hep-th/0605108];
C.A. Scrucca, *Soft masses in superstring models with anomalous* U(1) *symmetries*, *JHEP* **12** (2007) 092 [arXiv:0710.5105].

[21] Y. Kawamura and T. Kobayashi, *Soft scalar masses in string models with anomalous* U(1) *symmetry*, *Phys. Lett.* **B 375** (1996) 141 [*Erratum ibid.* **388** (1996) 867] [hep-ph/9601365];
P. Binetruy and E. Dudas, *Gaugino condensation and the anomalous* U(1), *Phys. Lett.* **B 389** (1996) 503 [hep-th/9607172];
G.R. Dvali and A. Pomarol, *Anomalous* U(1) *as a mediator of supersymmetry breaking*, *Phys. Rev. Lett.* **77** (1996) 3728 [hep-ph/9607383];
Y. Kawamura and T. Kobayashi, *Generic formula of soft scalar masses in string models*, *Phys. Rev.* **D 56** (1997) 3844 [hep-ph/9608233];
N. Arkani-Hamed, M. Dine and S.P. Martin, *Dynamical supersymmetry breaking in models with a Green-Schwarz mechanism*, *Phys. Lett.* **B 431** (1998) 329 [hep-ph/9803432].

[22] S. Chiossi and S. Salamon, *The intrinsic torsion of* SU(3) *and G_2 structures*, math.DG/0202282.

[23] S. Gurrieri, J. Louis, A. Micu and D. Waldram, *Mirror symmetry in generalized Calabi-Yau compactifications*, *Nucl. Phys.* **B 654** (2003) 61 [hep-th/0211102];
M. Graña, J. Louis and D. Waldram, *Hitchin functionals in $N = 2$ supergravity*, *JHEP* **01** (2006) 008 [hep-th/0505264].

[24] G. Lopes Cardoso, G. Curio, G. Dall'Agata, D. Lust, P. Manousselis and G. Zoupanos, *Non-Kähler string backgrounds and their five torsion classes*, *Nucl. Phys.* **B 652** (2003) 5 [hep-th/0211118].

[25] J. Werner, *Kleine Auflösungen spezieller dreimdimensionaler Varietäten*, Dissertation, University of Bonn, Germany (1987).

[26] P. Candelas, X.C. De La Ossa, P.S. Green and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, *Nucl. Phys.* **B 359** (1991) 21.

[27] W.-Y. Chuang, S. Kachru and A. Tomasiello, *Complex/symplectic mirrors*, *Commun. Math. Phys.* **274** (2007) 775 [hep-th/0510042].

[28] T.W. Grimm and J. Louis, *The effective action of $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys.* **B 699** (2004) 387 [hep-th/0403067];
T.W. Grimm, *The effective action of type-II Calabi-Yau orientifolds*, *Fortschr. Phys.* **53** (2005) 1179 [hep-th/0507153].

[29] I. Benmachiche and T.W. Grimm, *Generalized $N = 1$ orientifold compactifications and the Hitchin functionals*, *Nucl. Phys.* **B 748** (2006) 200 [hep-th/0602241].

[30] P. Koerber and L. Martucci, *From ten to four and back again: how to generalize the geometry*, *JHEP* **08** (2007) 059 [arXiv:0707.1038].

[31] D. Robbins and T. Wrase, *D-terms from generalized NS-NS fluxes in type II*, *JHEP* **12** (2007) 058 [arXiv:0709.2186].

[32] M. Karoubi and C. Leruste, *Algebraic topology via differential geometry*, Cabridge University Press, Cambridge U.K. (1987).

[33] S.A. Abel, M.D. Goodsell, J. Jaeckel, V.V. Khoze and A. Ringwald, *Kinetic mixing of the photon with hidden* U(1)*s in string phenomenology*, *JHEP* **07** (2008) 124 [arXiv:0803.1449].

[34] N.J. Hitchin, *Stable forms and special metrics*, math.DG/0107101; *Generalized Calabi-Yau manifolds*, *Quart. J. Math. Oxford Ser.* **54** (2003) 281 [math.DG/0209099].

[35] A. Strominger, *Massless black holes and conifolds in string theory*, *Nucl. Phys.* **B 451** (1995) 96 [hep-th/9504090].

[36] See, for example, K.A. Intriligator, P. Kraus, A.V. Ryzhov, M. Shigemori and C. Vafa, *On low rank classical groups in string theory, gauge theory and matrix models*, *Nucl. Phys.* **B 682** (2004) 45 [hep-th/0311181], and references therein.

[37] M. Aganagic, C. Beem, J. Seo and C. Vafa, *Extended supersymmetric moduli space and a SUSY/non-SUSY duality*, arXiv:0804.2489.

[38] L. Hollands, J. Marsano, K. Papadodimas and M. Shigemori, *Nonsupersymmetric flux vacua and perturbed* $N = 2$ *systems*, arXiv:0804.4006.

[39] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[40] H. Jockers and J. Louis, *The effective action of D*7*-branes in* $N = 1$ *Calabi-Yau orientifolds*, *Nucl. Phys.* **B 705** (2005) 167 [hep-th/0409098]; *D-terms and F-terms from D*7*-brane fluxes*, *Nucl. Phys.* **B 718** (2005) 203 [hep-th/0502059].

[41] M. Haack, D. Krefl, D. Lüst, A. Van Proeyen and M. Zagermann, *Gaugino condensates and D-terms from D*7*-branes*, *JHEP* **01** (2007) 078 [hep-th/0609211].

[42] M. Buican, D. Malyshev, D.R. Morrison, H. Verlinde and M. Wijnholt, *D-branes at singularities, compactification and hypercharge*, *JHEP* **01** (2007) 107 [hep-th/0610007].

[43] W. Barth, C. Peters and A. Van den Ven, *Compact complex surfaces*, Springer-Verlag, Berlin Germany (1984).

[44] M. Demazure, *Surfaces de Del Pezzo, II, III, IV et V*, in *Séminaire sur les singularités des surface*, Lecture Notes in Mathematics vol. 777, Springer-Verlag, Berlin Germany (1980).

[45] F. Cachazo, K.A. Intriligator and C. Vafa, *A large-N duality via a geometric transition*, *Nucl. Phys.* **B 603** (2001) 3 [hep-th/0103067].

[46] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[47] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and* χ_{SB}*-resolution of naked singularities*, *JHEP* **08** (2000) 052 [hep-th/0007191].

[48] S.B. Giddings and A. Maharana, *Dynamics of warped compactifications and the shape of the warped landscape*, *Phys. Rev.* **D 73** (2006) 126003 [hep-th/0507158];
C.P. Burgess et al., *Warped supersymmetry breaking*, *JHEP* **04** (2008) 053 [hep-th/0610255].

[49] M.R. Douglas, J. Shelton and G. Torroba, *Warping and supersymmetry breaking*, arXiv:0704.4001.

[50] G. Shiu, G. Torroba, B. Underwood and M.R. Douglas, *Dynamics of warped flux compactifications*, *JHEP* **06** (2008) 024 [arXiv:0803.3068].

[51] O. DeWolfe, S. Kachru and M. Mulligan, *A gravity dual of metastable dynamical supersymmetry breaking*, *Phys. Rev.* **D 77** (2008) 065011 [arXiv:0801.1520].

[52] S. Kachru, J. Pearson and H.L. Verlinde, *Brane/flux annihilation and the string dual of a non- supersymmetric field theory*, *JHEP* **06** (2002) 021 [hep-th/0112197].

[53] M. Reid, *Young person's guide to canonical singularities*, *Proc. Symp. Pure Math.* **46** (1987) 345.

[54] F. Cachazo, B. Fiol, K.A. Intriligator, S. Katz and C. Vafa, *A geometric unification of dualities*, *Nucl. Phys.* **B 628** (2002) 3 [hep-th/0110028].

[55] D.-E. Diaconescu, B. Florea, S. Kachru and P. Svrček, *Gauge-mediated supersymmetry breaking in string compactifications*, *JHEP* **02** (2006) 020 [hep-th/0512170];
D.-E. Diaconescu, R. Donagi and B. Florea, *Metastable quivers in string compactifications*, *Nucl. Phys.* **B 774** (2007) 102 [hep-th/0701104].

[56] L. Bayle and A. Beauville, *Birational involutions of* $\mathbb{P}^2$, *Asian J. Math.* **4** (2000) 11 [math.AG/9907028].

[57] I.V. Dolgachev and V.A. Iskovskikh, *Finite subgroups of the plane Cremona group*, math.AG/0610595;
I.V. Dolgachev, *Reflection groups in algebraic geometry*, to appear in the *Bull. A.M.S.*, math.AG/0610938.

[58] T.W. Grimm, *Non-perturbative corrections and modularity in* $N = 1$ *type IIB compactifications*, *JHEP* **10** (2007) 004 [arXiv:0705.3253].

[59] D.R. Morrison and C. Vafa, *Compactifications of F-theory on Calabi-Yau Threefolds — II*, *Nucl. Phys.* **B 476** (1996) 437 [hep-th/9603161].

[60] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [hep-th/0502058].

[61] P. Candelas, X. De La Ossa, A. Font, S.H. Katz and D.R. Morrison, *Mirror symmetry for two parameter models. I*, *Nucl. Phys.* **B 416** (1994) 481 [hep-th/9308083]; P. Candelas, A. Font, S.H. Katz and D.R. Morrison, *Mirror symmetry for two parameter models. II*, *Nucl. Phys.* **B 429** (1994) 626 [hep-th/9403187].

[62] F. Hirzebruch and J. Werner, *Some examples of threefolds with trivial canonical bundle*, SFB.MPI 85-58 (1985).

[63] S. Mori and S. Mukai, *On Fano 3-folds with* $B_2 \geq 2$, in *Algebraic varieties and analytic varieties*, Advanced Studies in Pure Mathematics volume 1, S. Iitaka ed., Kinokuniya, Tokyo Japan (1983).

[64] K. Saito, *Quasihomogene isolierte Singularitäten von Hyperflächen*, *Invent. Math.* **14** (1971) 123.

[65] D. Malyshev, *Del Pezzo singularities and SUSY breaking*, arXiv:0705.3281.

[66] S. Hosono, A. Klemm, S. Theisen and S.-T. Yau, *Mirror symmetry, mirror map and applications to Calabi-Yau hypersurfaces*, *Commun. Math. Phys.* **167** (1995) 301 [hep-th/9308122]; *Mirror symmetry, mirror map and applications to complete intersection Calabi-Yau spaces*, *Nucl. Phys.* **B 433** (1995) 501 [hep-th/9406055].

[67] R. Slansky, *Group theory for unified model building*, *Phys. Rept.* **79** (1981) 1.

[68] R.W. Carter, *Conjugacy classes in the Weyl group*, *Compos. Math.* **25** (1972) 1.

[69] M. Aganagic, C. Beem and S. Kachru, *Geometric transitions and dynamical SUSY breaking*, *Nucl. Phys.* **B 796** (2008) 1 [arXiv:0709.4277].

[70] R. Blumenhagen, S. Moster and E. Plauschinn, *Moduli stabilisation versus chirality for MSSM like type IIB orientifolds*, *JHEP* **01** (2008) 058 [arXiv:0711.3389].

[71] D.A. Cox and S. Katz, *Mirror symmetry and algebraic geometry*, American Mathematical Society, Providence U.S.A. (2000), pg. 469.

[72] S.H. Katz, A. Klemm and C. Vafa, *M-theory, topological strings and spinning black holes*, *Adv. Theor. Math. Phys.* **3** (1999) 1445 [hep-th/9910181].

[73] M. Kreuzer, *Calabi Yau data*, http://hep.itp.tuwien.ac.at/˜kreuzer/CY/.

[74] K. Kodaira, *On compact analytic surfaces, II*, *Annals of Math.* **77** (1963) 563; *On compact analytic surfaces, III*, *Annals of Math.* **78** (1963) 1.

[75] A. Klemm, B. Lian, S.S. Roan and S.-T. Yau, *Calabi-Yau fourfolds for M- and F-theory compactifications*, *Nucl. Phys.* **B 518** (1998) 515 [hep-th/9701023].

[76] J. Louis, J. Sonnenschein, S. Theisen and S. Yankielowicz, *Non-perturbative properties of heterotic string vacua compactified on $K3 \times T^2$*, *Nucl. Phys.* **B 480** (1996) 185 [hep-th/9606049].

[77] A. Klemm, P. Mayr and C. Vafa, *BPS states of exceptional non-critical strings*, hep-th/9607139.